Addictive Platforms

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October 31, 2020

Abstract

We study platform competition for consumer attention, in which platforms choose the addictiveness of their services. A more addictive platform yields a lower service utility, but a higher marginal utility for consumers of allocating their attention. In equilibrium, platforms choose inefficiently high addictiveness, even though collectively reducing the addictiveness would benefit consumers without hurting platforms. A digital curfew—i.e., limiting the amount of attention consumers can spend on platforms—may benefit consumers by reducing the equilibrium addictiveness. Platforms decrease addictiveness when they can charge consumers for their services. However, consumers may prefer free addictive platforms to nonaddictive platforms that charge positive prices.

JEL codes: D4, L1

Keywords: platform competition, addiction, attention, merger

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1 Introduction

Online platforms, such as Facebook, Google, and Twitter, monetize consumer attention by selling advertising spaces. Because attention is a scarce resource, competition may encourage these firms to offer better services that attract consumers. However, there is a growing concern for consumers and policymakers—that competition for attention may also incentivize these firms to design their services to increase consumers’ attention potentially at the expense of their welfare. For example, a platform may adopt algorithmic news feeds that display news users are most likely to click, even if the quality of the news is low on other dimension, such as accuracy; the platform may also adopt a user interface, such as a notification system or infinite scrolling, to increase user attention (Scott Morton et al., 2019).\(^1\)

This paper studies competition between platforms for consumer attention, in which platforms choose the “addictiveness” of their services. In our model, a more addictive platform yields a lower service utility but a higher marginal utility for consumers of allocating their attention (Section 2.1 motivates this formulation). As a result, consumers prefer to join less addictive platforms, but once they join, they allocate a greater share of attention to more addictive platforms than to less addictive ones. We allow consumers to be naive—i.e., they may underestimate the addictiveness of a platform when they decide whether to join it.

There are three main findings. First, the equilibrium is inefficient: All platforms choose positive addictiveness, even though collectively reducing addictiveness would increase consumer surplus without affecting platform profit. Also, in a less competitive market, platforms choose higher addictiveness, which leads to lower efficiency and consumer welfare: When a platform faces only a few rivals, it can set high addictiveness to capture consumer attention without deterring their participation, because consumers do not have many other services to use. The effect is stronger when consumers are naive. Similarly, a platform merger increases the addictiveness of merged services, and harms consumers and efficiency.

Second, the business models of platforms affect their addictiveness. Specifically, we compare

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\(^1\)For example, Scott Morton and Dinielli (2020) argue that “another reduction in quality that Facebook’s market power allows is the serving of addictive and exploitative content to consumers. Facebook deploys various methods to maintain user attention—so that it can serve more ads—using techniques that the medical literature has begun to demonstrate are potentially addictive.”
the equilibria of two games: One is the baseline model in which platforms earn revenue from consumer attention. The other is a model in which platforms earn revenue from monetary prices they charge on consumers (e.g., subscription fees). We find that price competition leads to lower addictiveness and a higher total surplus. The result supports the intuition that advertising-supported companies may have a stronger incentive to adopt features that keep consumers’ attention (e.g., providing catchy but inaccurate content) than platforms that do not monetize attention. However, the impact of different business models on consumers depends on their naivete. For example, sophisticated consumers are better off under competition for attention, in which they can access free services with positive addictiveness. The business model also affects the impact of a merger: A merger can reduce efficiency more under competition for attention than under price competition.

Finally, we examine the impact of a digital curfew, which exogenously limits the total amount of attention consumers can spend on digital services. In our model, the cost of a digital curfew is to restrict consumers’ choice, and the benefit is to incentivize platforms to reduce their addictiveness. We show that if the degree of consumers’ naivete is high, the benefit dominates the cost. As a result, a digital curfew can benefit naive consumers.

**Related literature** First, this paper contributes to the nascent literature on possible negative impacts of digital services on consumers (Allcott and Gentzkow, 2017; Allcott et al., 2020; Mosquera et al., 2020). A recent discussion points out that technology companies may have an incentive to adopt features (e.g., user interfaces) that increase user engagement at the expense of their welfare (Alter, 2017; Scott Morton et al., 2019; Newport, 2019; Rosenquist et al., 2020). We contribute to this literature by examining possible interactions between competition for attention and the addictiveness of digital services. Although we later motivate our model based on habit formation with a time-inconsistent agent, we largely abstract away from dynamics and behavioral biases relevant to addiction (Becker and Murphy, 1988; Gruber and Köszegi, 2001; Orphanides and Zervos, 1995). This simplification enables us to study various regulations and business models.

Second, our paper is related to the literature on competition for attention (Wu, 2017; Evans, 2020). For example, in 2003 Thailand implemented a shutdown law that banned young people from playing online games between 22:00 and 06:00. In 2011, South Korea passed a similar legislature, known as the Youth Protection Revision Act. In 2007, China introduced the so-called “fatigue” system under which game developers need to reduce or stop giving out rewards (e.g., game items, experience value) in games after a player reached a certain hours of play.
2017, 2019; Prat and Valletti, 2019; Galperti and Trevino, 2020) as well as regulation against dominant platforms that monetize attention (Crémer et al., 2019; U.K. Digital Competition Expert Panel, 2019; Scott Morton and Dinielli, 2020). Several papers, such as Bordalo et al. (2016) and Anderson and De Palma (2012), also study competition for attention. More broadly, the literature on platform competition is relevant to competition for attention (e.g., Rochet and Tirole 2003; Anderson and Coate 2005; Armstrong 2006). Relative to this literature, our model has two new features. First, a consumer in our model decides not only whether to join a platform but also how much attention to allocate. This feature renders both a platform’s service utility and a marginal utility (of allocating attention) relevant. Second, a platform can choose higher addictiveness to decrease the service utility and increase marginal utility. The choice of addictiveness has a different welfare implication compared to the choice of price or quality.3

Roadmap Section 2 sets up the model and provides some applications. Section 3 characterizes the unique equilibrium, and Section 4 compares it to the one in a model of price competition. Section 5 examines the impact of a digital curfew, and Section 6 studies platform merger. Section 7 discusses the cases of heterogeneous consumers and monopoly, and Section 8 concludes.

2 Model

There are $K \geq 2$ platforms and a single consumer.4 We write $K$ for the number and the set of the platforms. The consumer (she) has $A > 0$ units of attention. Suppose the consumer joins platform $k \in K$ and allocates $a_k \in [0, A]$ units of attention to it. Then, platform $k$ receives a payoff of $a_k$. The consumer’s utility from platform $k$ is $u(a_k, d_k)$, where $d_k \in \mathbb{R}_+$ is the addictiveness of platform $k$. We impose the following assumptions (see Figure 1).

Assumption 1. The function $u(\cdot, \cdot) : \mathbb{R}_+^2 \to \mathbb{R}$ is twice continuously differentiable and satisfies the following:

3Relatedly, De Corniere and Taylor (2020) study the role of data in competition in utilities framework. There, data affect the marginal incentives of firms to offer utilities to consumers. Choi and Jeon (2020) study the incentive of platforms to adopt a technology that shifts surplus from one side to the other.

4The model is equivalent to the one with a continuum of identical consumers.
(a) **Monotonicity/Concavity:** For any \( d \geq 0 \), \( u(a, d) \) is strictly increasing and concave in \( a \).

(b) **Negativity:** For each \( a \geq 0 \), \( u(a, d) \) is strictly decreasing in \( d \) and negative for some \( d \).

(c) **Complementarity:** For each \( a \geq 0 \), \( \frac{\partial u}{\partial a}(a, d) \) is strictly increasing in \( d \).

(d) **No harm from zero addictiveness:** \( u(0, 0) \geq 0 \).

![Figure 1: Utilities under \( d_L \) and \( d_H > d_L \).](image)

Points (b) and (c) imply that higher addictiveness decreases the consumer’s utility from joining a platform but increases her marginal utility of allocating attention. **Assumption 1** holds if, for example, \( u(a, d) = (1 + d) \log(1 + a) - cd \) with \( c > \log(1 + A) \), or \( u(a, d) = \hat{u}(a - d) \) with an increasing and concave \( \hat{u}(\cdot) \). **Section 2.1** provides discussion and applications.

Before joining platforms, the consumer may underestimate the addictiveness of platforms. Formally, the consumer is characterized by an exogenous parameter \( s \in (0, 1] \), which captures the degree of sophistication. If a platform chooses addictiveness \( d \), the consumer perceives it as \( s \cdot d \) upon deciding which platforms to join. The consumer is **sophisticated** if \( s = 1 \) and **naive** if \( s < 1 \). **Section 7** discusses the case in which consumers have heterogeneous \( s \).

The timing of the game is as follows: First, each platform \( k \in K \) simultaneously chooses its addictiveness, \( d_k \geq 0 \). Second, given the perceived addictiveness \( (sd_k)_{k \in K} \), the consumer chooses the set \( \hat{K} \subset K \) of platforms to join. In equilibrium, \( \hat{K} \) maximizes the perceived indirect utility.
\( V(K') \) across all \( K' \in 2^K \), where

\[
V(K') := \max_{(a_k)_{k \in K'}} \sum_{k \in K'} u(a_k, s d_k) \tag{1}
\]

\[
s.t. \sum_{k \in K'} a_k \leq A \quad \text{and} \quad a_k \geq 0, \forall k \in K'.
\]

The constraints mean that the consumer cannot spend more than \( A \) amount of attention or negative amount of attention. If \( \hat{K} = \emptyset \), all players obtain a payoff of zero, and the game ends. After joining platforms \( \hat{K} \neq \emptyset \), the consumer observes the true addictiveness of each platform, then allocates her attention. In equilibrium, the consumer solves

\[
\max_{(a_k)_{k \in \hat{K}}} \sum_{k \in \hat{K}} u(a_k, d_k) \tag{2}
\]

\[
s.t. \sum_{k \in \hat{K}} a_k \leq A \quad \text{and} \quad a_k \geq 0, \forall k \in \hat{K}.
\]

The above two maximization problems coincide if \( s = 1 \).

Our solution concept is pure-strategy subgame perfect equilibrium (SPE), which we call “equilibrium.”\(^5\) When we evaluate welfare such as consumer surplus and total surplus (i.e., the sum of payoffs of all players), we use the true addictiveness \((d_k)_{k \in K}\).

### 2.1 Discussion on Modeling Addictive Platforms

The literature formulates addiction as a dynamic process, such as habit formation (e.g., Becker and Murphy 1988). In contrast, we study addictive digital services in a static model. To reconcile this tension, we motivate our \( u(a, d) \) in a framework that reflects a dual-self model with habit formation.\(^6\) The framework is more of an example than a microfoundation, but it helps us understand the implicit relation between the addictiveness of a platform and the consumer’s addiction.

For simplicity, assume that the consumer correctly perceives the addictiveness of platforms

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\(^5\)Even if \( s < 1 \), we can use SPE by treating the consumer who solves (1) and the consumer who solves (2) as different players who have different objectives.

\(^6\)The combination of time-inconsistent preferences and habit formation also appear, for example, in Gruber and Köszegi (2001) in a much more general way.
Given addictiveness \((d_1, \ldots, d_K)\), consider the following three-period problem (see Figure 2). In \(t = 1\), the consumer chooses the set \(K' \subset K\) of platforms to join. In \(t = 2\), the consumer exogenously allocates attention \(a_0 > 0\) and obtains utility \(u_0 \geq 0\) on each platform in \(K'\). This period is a “pre-addiction” stage—i.e., the consumer has yet to be addicted, and thus the service utilities or the optimal amount of attention to allocate does not depend on \((d_1, \ldots, d_K)\).\(^7\) In \(t = 3\), the consumer endogenously allocates her attention \(A\) across platforms \(K'\). This period is a “post-addiction” stage: If the consumer allocates attention \(a\) to platform \(k\), her payoff is \(\hat{u}(a - a_0d_k)\), where \(\hat{u}(\cdot)\) is an increasing concave function with \(\hat{u}(0) \geq 0\). This functional form captures linear habit formation (e.g., Rozen 2010). Here, \(a_0d_k\) is the reference point against which the consumer evaluates service consumption of platform \(k\) in \(t = 3\). In this framework, we can interpret \(\frac{1}{d_k}\) as the “rate of disappearance of the physical and mental effects of past consumption” (Becker and Murphy, 1988). For a given amount of attention in \(t = 2\), a higher \(d_k\) imposes a greater harm on the consumer in \(t = 3\). As a result, she needs to increase her attention in \(t = 3\) to ensure the same payoff.

Motivated by a dual-self model (e.g., Thaler and Shefrin 1981; Fudenberg and Levine 2006), we assume that the long-run self makes the participation decision, and the short-run self allocates attention. As shown above, in \(t = 2\) and \(t = 3\), the short-run self myopically allocates attention. In particular, at the post addiction stage, she allocates attention \((a^*_k)_{k \in K'}\) to maximize \(\sum_{k \in K'} \hat{u}(a_k - a_0d_k)\). However, in \(t = 1\), the long-run self decides which platforms to join, anticipating the behavior of short-run selves in \(t = 2\) and \(t = 3\). Assume the long-run self has discount factor \(\delta\). Then, the consumer’s participation decision is based on \(u(a_k, d_k) := u_0 + \delta\hat{u}(a_k - a_0d_k)\), which satisfies Assumption 1.

The above framework clarifies that our model with a high \(s\) is suitable when a consumer is susceptible to addictive features of online services, but she recognizes it and may avoid joining platforms as a commitment device. In contrast, a low \(s\) means that the consumer is susceptible to addictive features and does not recognize it. The model is not suitable for a consumer who joins platforms but can use them cautiously to avoid addiction. Such a situation would correspond to the consumer who is forward-looking in \(t = 2, 3\).

\(^7\)Here, to derive our functional form, we assume that \(a_0\) does not depend on the number of platforms the consumer has joined. One way to endogenize this behavior is to assume that the consumer’s utility from each platform in \(t = 2\) is \(v(a)\), which is maximized at an interior optimum \(a_0\) such that \(K A_0 \leq A\).
Other than digital addiction, our model can capture the choice of a platform that increases the marginal utility of allocating attention at the expense of service quality. Below, we present two examples.

**Example 1 (Data collection and personalization).** A platform requests consumers to provide their personal data upon participation. Let \( d \) denote the amount of data the platform requests. To provide data, consumers incur a privacy cost—e.g., the risk of data leakage, identity theft, and discrimination. Suppose consumers incur a linear privacy cost, \( c \cdot d \). The platform can use their data to personalize offerings, which increases the value of the service from the base value \( w(a) \) to \((1 + d)w(a)\), where \( w(\cdot) \) is increasing and concave. Then, a consumer’s utility from joining the platform is \( u(a, d) := (1+d)w(a)−cd \). For \( c > w(A) \), \( u(a, d) \) satisfies Assumption 1. If consumers join platforms but do not use the services, they may receive a negative utility (i.e., \( u(0, d) < 0 \) for a large \( d \)), because they experience the potential downside of data collection without enjoying the service.

**Example 2 (Offering tempting content).** Scott Morton et al. (2019) provide an example in which a video streaming platform recommends videos that lead viewers towards false or dangerous content. A higher \( d \) can correspond to a platform that offers such content, which has lower-quality but captures consumer attention. In our model, the consumer may receive a negative utility if she joins a platform with \( d > 0 \) but allocates no attention. We may understand such a loss as the cost.

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\[ u_0 + \delta \cdot \hat{u}(a_k - d_ka_0) \]
\[ := u(a_k, d_k) \]

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Scott Morton et al. (2019) describes the situation as follows: “More disturbing examples of low-quality content are YouTube recommended videos that lead the viewer towards false or dangerous content. Prior to having these patterns made public and criticized, a Google search about the earth’s geology would lead to a chain of recommendations that resulted in ‘flat earth’ content; YouTube would offer teenage girls interested in diets videos about how to get anorexia, and so forth.”
of self-control, which the consumer incurs when they know tempting content is available, but do not consume it (e.g., Gul and Pesendorfer 2001).

In practice, platforms may also introduce some features that increase consumer attention without reducing their welfare. We abstract away from such beneficial features, because it is less of concern from welfare perspective. Indeed, we may understand $u(a, d)$ as the consumer’s utility conditional on that the platform has adopted all features that increase consumers’ marginal utility without lowering their welfare.\(^9\)

### 2.2 Other Modeling Assumptions

**Platform’s revenue.** A platform’s profit equals the amount of attention $a_k$ from the consumer. However, all the results continue to hold (with the same proof) in the following setting: If the consumer allocates attention $(a_1, \ldots, a_K)$, platform $k$ earns a payoff of $r_k(a_1, \ldots, a_K)$ that is strictly increasing in $a_k$ and may depend arbitrarily on $(a_j)_{j \in K \setminus \{k\}}$. For example, a platform’s payoff captures its revenue in the advertising market, in which platforms can sell consumer attention at a market price. Alternatively, more attention may lead to a larger revenue from ancillary services such as in-app purchases. To facilitate the analysis, we assume that addictiveness affects platform’ revenues only through the consumer’s attention allocation.

**Allocation of attention.** In our model, the maximum supply of attention is inelastic at $A$, and the consumer’s utility from a service is increasing in $a$.\(^{10}\) As a result, the consumer allocates all of her attention $A$ to platforms she has joined. A more general model could consider an elastic supply of attention.

### 3 Equilibrium

First, we present the equilibrium (see Appendix A for the proof).

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\(^9\)Formally, suppose the consumer’s utility of joining a platform is $u(a, d, b)$, which satisfies Assumption 1 with respect to $(a, d)$ and increases in $b \in [0, 1]$. Then we can define $u(a, d) = u(a, d, 1)$.

\(^{10}\)The idea that attention is a finite resource is not new—e.g., Simon (1971) and Davenport and Beck (2001).
Proposition 1. The unique equilibrium is inefficient: Each platform sets positive addictiveness \( d_s(K) := \frac{d(K)}{s} \), where \( d(K) > 0 \) solves the equation

\[
K \cdot u\left( \frac{A}{K}, d(K) \right) = (K - 1) \cdot u\left( \frac{A}{K-1}, d(K) \right).
\] (3)

The consumer allocates her attention equally across all platforms. The equilibrium addictiveness \( d_s(K) \) is strictly decreasing in \( s \) and \( K \), and \( \lim_{K \to \infty} d_s(K) = 0 \). Consumer surplus and total surplus are strictly increasing in \( K \) and \( s \).

Proposition 1 states the equilibrium is inefficient, because platforms choose positive addictiveness: If all platforms choose \( d = 0 \), their profits remain the same and the consumer’s payoff increases, and thus the outcome is Pareto-improvement. The result also implies that platforms choose higher addictiveness when they face fewer competitors. As a result, the lack of competition harms consumers and efficiency through the lower variety of services and their higher addictiveness. The impact of a lower \( K \) on addictiveness is more pronounced when the consumer is naive.

The intuition is as follows. Upon choosing addictiveness, a platform faces a trade-off. On the one hand, higher addictiveness renders its service less attractive to consumers. On the other hand, conditional on joining, consumers allocate more attention to more addictive services, because of the complementarity between attention and addictiveness. Thus, each platform prefers to increase the addictiveness so long as the consumer joins it. As a result, the equilibrium addictiveness makes the consumer indifferent between joining and not joining each platform. For the sophisticated consumer, the equilibrium addictiveness satisfies the indifference condition (3). For the naive consumer, the addictiveness is scaled up by \( \frac{1}{s} \), i.e., the extent to which consumers underestimate the addictiveness of services.

To see the comparative statics in \( K \), consider the consumer’s trade-off. The consumer’s loss of not joining a platform (say \( k \)) is that she does not receive its service utility. The benefit is that she can allocate the saved attention to other platforms \( j \neq k \). When there are a fewer platforms, this benefit of reallocating the saved attention is smaller, because the consumer will allocate a larger amount of attention to each platform \( j \neq k \), but the marginal utility of allocating attention

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11In our model, platforms do not incur costs to raise addictiveness. If platforms incur costs to increase addictiveness, not only the consumer but all platforms will be strictly better off under the outcome with zero addictiveness than the one with positive addictiveness.
is decreasing. As a result, for a small $K$ the consumer incurs a higher net loss of not joining a platform. Thus, platform can set higher equilibrium addictiveness without deterring consumer participation. However, because all platforms increase their addictiveness, it harms consumers without changing her allocation of attention across platforms.

Finally, the equilibrium addictiveness is decreasing in the degree of consumer sophistication, $s$. Thus, educating consumers about addictive features of digital platforms can enhance consumer welfare. In our model, raising $s$ benefits consumers not because they change how they use platforms, but because platforms decrease their addictiveness.

## 4 Competition on Prices and Addictiveness

We now examine how the business models of platforms affect the equilibrium addictiveness and welfare. Specifically, we compare the above model (i.e., competition for attention) with a model of price competition, in which platforms earn revenue from monetary prices, such as subscription fees.

Formally, we define the game of price competition as follows. First, each platform $k \in K$ simultaneously chooses its addictiveness $d_k \geq 0$ and price $p_k \in \mathbb{R}$. The consumer observes $(d_k, p_k)_{k \in K}$, then chooses the set $K' \subset K$ of platforms to join. Then the consumer chooses $(a_k)_{k \in K'}$, subject to the constraint that $a_k \geq 0$ and $\sum_{k \in K'} a_k \leq A$. In equilibrium, she solves the participation problem (1) with $u(a_k, s d_k)$ replaced by $u(a_k, s d_k) - p_k$ (the attention allocation problem (2) remains the same because the prices are sunk). Each platform $k \in K'$ receives a payoff of $p_k$, and any platform $k \not\in K'$ obtains a payoff of zero. The consumer receives a payoff of $\sum_{k \in K'} [u(a_k, d_k) - p_k]$ and zero if $K' \neq \emptyset$ and $K' = \emptyset$, respectively.

The platforms have different payoff functions under the two modes of competition. In particular, under price competition, platforms can charge consumers for their services, but they do not monetize consumer attention. This model captures business models that are not supported by advertising and charge subscription fees, such as Netflix, Spotify, and Youtube Premium.\textsuperscript{12}

The following result compares competition for attention and price competition (see Appendix

\textsuperscript{12}It is beyond the scope of this paper to consider the endogenous choice of business models, or a business model that offers both ad-supported and subscription plans.
Proposition 2. Fix any \( s \in (0, 1] \). Under price competition, there is a unique equilibrium, in which all platforms choose zero addictiveness and set a price of 
\[
p^* = K u \left( \frac{A}{K}, 0 \right) - (K - 1) u \left( \frac{A}{K-1}, 0 \right).
\]
Also, the following holds.

1. Total surplus is higher under price competition than under competition for attention.

2. There is an \( s^* \in (0, 1) \) such that the consumer is better off under price competition than competition for attention if and only if \( s \leq s^* \). This welfare comparison is strict if \( s \neq s^* \).

Proposition 2 consists of several points. First, when platforms compete on prices, the equilibrium involves zero addictiveness. The intuition is simple: The revenue of a platform does not depend on attention, so each platform prefers to reduce addictiveness and increase the price. As a result, all platforms set zero addictiveness, and the equilibrium price \( p^* \) makes the consumer indifferent between joining and not joining each platform.

Second, total surplus is higher when platforms compete on prices, because monetary payment transfers surplus but does not distort service quality. Thus, free platforms might come with a hidden social cost of higher addictiveness.

Third, although price competition is more efficient, it may harm consumers. In particular, a sophisticated consumer is better off when platforms earn revenue only from attention. To see the intuition, suppose that the consumer is sophisticated, i.e., \( s = 1 \). Let \( d^* \) denote the equilibrium addictiveness under competition for attention. When platforms compete for attention, the consumer’s \textit{per platform} payoff is 
\[
u \left( \frac{A}{K}, d^* \right),
\]
which we can rewrite as follows:
\[
u \left( \frac{A}{K}, d^* \right) = (K - 1) \left[ u \left( \frac{A}{K-1}, d^* \right) - u \left( \frac{A}{K}, d^* \right) \right].
\] (4)
The right-hand side captures the gain of refusing to join a platform and allocating the saved attention to other \( K - 1 \) platforms. The bracketed term is the gain of reallocation for each of \( K - 1 \) platforms, which is multiplied by the number of the platforms, \( K - 1 \). In equilibrium, this gain is equal to the loss of not joining a platform, which is the left-hand side. This trade-off also works in
the model of price competition:

\[ u \left( \frac{A}{K}, 0 \right) - p^* = (K - 1) \left[ u \left( \frac{A}{K - 1}, 0 \right) - u \left( \frac{A}{K}, 0 \right) \right]. \tag{5} \]

The right-hand side of (4) is larger than that of (5), because \( \frac{\partial u}{\partial a} \) is increasing in \( d \)—i.e., under attention competition, the consumer faces a higher gain of refusing to join a platform and stay with other \( K - 1 \) platforms. Comparing the left-hand sides, we can conclude that a sophisticated consumer is better off when platforms monetize attention. As the degree of naivite (i.e., \( 1/s \)) increases, the consumer’s payoff under competition for attention decreases, whereas her payoff under price competition remains the same. Thus for a small \( s \), the consumer is better off under price competition, in which she pays positive prices to use nonaddictive services.

Proposition 2 supports the idea that ad-financed platforms may have an incentive to “keep users online for another minute in order to show more ads,” possibly at the expense of consumer welfare (Scott Morton et al., 2019). At the same time, the result suggests that which business models attain a higher consumer surplus could depend on other factors, such as consumer sophistication.

5 Digital Curfew

A digital curfew restricts the amount of time consumers can spend on digital services. For example, the Social Media Addiction Reduction Technology Act (the “SMART” Act) proposed in the US aims at curbing social media addiction by requesting that social media companies automatically limit the amount of time a user may spend.\(^{13}\) In our model, a digital curfew reduces \( A \). The following lemma presents the impact of a digital curfew on the equilibrium addictiveness. Unless otherwise noted, the results of this section concern competition for attention (see Appendix C for the proof).

**Lemma 1.** The equilibrium addictiveness is increasing in the total amount of attention, \( A \).

The intuition is as follows. The equilibrium addictiveness makes the consumer indifferent between joining and not joining each platform, e.g., \( K u \left( \frac{A}{K}, d^* \right) = (K - 1)u \left( \frac{A}{K - 1}, d^* \right) \). Because

the consumer faces a decreasing marginal utility of allocating attention, she gains more from a marginal increase of $A$ when she joins more platforms, e.g., $K u \left( \frac{A+\Delta}{K}, d^* \right) > (K-1) u \left( \frac{A}{K-1}, d^* \right)$ for a small $\Delta > 0$. Thus, platforms can increase addictiveness without deterring consumer participation.

Does the consumer benefit from a digital curfew? To see whether the consumer benefits from a lower $A$, let $U_s(A)$ denote the consumer’s equilibrium payoff given $(s, A)$. The following result shows that under certain conditions, a digital curfew benefits naive consumers but harms sophisticated consumers (see Appendix D for the proof).

**Proposition 3.** Suppose there is a strictly increasing and concave function $w(\cdot)$ such that $w(0) = 0$ and $u(a, d) = (1 + d) w(a) - cd$ for some $c > w(A)$. Then, the following holds.

1. There is an $s^* \in [0, 1]$ such that $U'_s(A) < 0$ and $U'_s(A) > 0$ if $s < s^*$ and $s > s^*$, respectively.

2. If $w(a) = \log(1 + a)$, the equilibrium payoff of a sophisticated consumer (i.e., $s = 1$) is increasing in $A$.

To see the intuition for Point 1, let $d(A)$ denote the equilibrium addictiveness given $A$ when $s = 1$. Then, we obtain

$$U'_s(A) = u_1 \left( \frac{A}{K}, \frac{d(A)}{s} \right) + Ku_2 \left( \frac{A}{K}, \frac{d(A)}{s} \right) \cdot \frac{d'(A)}{s}.$$

The first term captures the benefit of increasing $A$: With fixed addictiveness, a higher $A$ benefits the consumer, because $u(a, d)$ is increasing in $a$. The second term captures the negative impact: By Lemma 1, a higher $A$ leads to higher equilibrium addictiveness. The negative effect is more pronounced for a smaller $s$ (i.e., higher naivete). As a result, the overall effect is likely to be negative for a smaller $s$. In the proof, we use the functional form assumption to show that the benefit of a larger $A$ indeed dominates the cost if and only if $s$ is above a cutoff. With an additional assumption, we can show that any reduction of $A$ harms the sophisticated consumer (Point 2).

**Proposition 3** illustrates that putting a cap on $A$ may hurt consumers. This finding has policy implications. Policymakers need to be cautious in claiming that the digital curfew would universally make consumers better off, though such a regulatory action may be effective in lowering
addictiveness. Most of the digital curfews and shutdown laws are to protect young children from addiction to online games since they may not have self-control against temptation. The mechanism in our model is different. Policymakers may not consider how the regulation would work via platform competition. But, our analysis uncovers a potential mechanism of an unintended consequence in the paternalistic regulatory action.

**Remark 1 (Voluntary Reduction of \(A\)).** A related question is whether consumers can voluntarily implement a digital curfew. To see this, suppose that there is a continuum of consumers, each of whom \(i \in [0, 1]\) chooses the maximum amount of attention \(A_i \in [0, A]\) she can spend on platforms. After consumers choose \((A_i)_{i \in [0,1]}\), the original game of attention competition is played. In the unique equilibrium, all consumers choose the maximum attention \(A\), because each consumer is atomless and her choice does not affect the behavior of platforms. Thus, consumers cannot voluntarily enforce a digital curfew. However, Proposition 3 suggests that consumers could be better off by collectively reducing \(A_i\)’s.

### 6 Merger and Addictiveness

In this section, we examine the impact of platform merger on the equilibrium addictiveness and welfare. We assume that a merger does not change the number of services available in the market, but merged services are bundled—i.e., the consumer can choose between joining all of the merged services or joining none of them. We show that a merger harms efficiency when platforms monetize attention, but not if they earn revenue by charging consumers for their services.

Formally, we compare the original game to the *post-merger* game. We first describe the post-merge game under competition for attention. The game is characterized by a market structure \(\mathcal{M}\), which is a partition of \(\mathcal{K} := \{1, \ldots, K\}\). We write \(\mathcal{M} = \{P_1, \ldots, P_M\}\), where each \(P_m \subset \mathcal{K}\) is a merged platform that consists of platforms (or services) \(k \in P_m\). The original game corresponds to \(\mathcal{M} = \{\{k\}\}_{k \in \mathcal{K}}\). For notational simplicity, we write \(M\) for the set and the number of platforms in the post-merger game. Given any \(\mathcal{M}\), the post-merger game works as follows.

1. Each platform \(m \in M\) simultaneously chooses its addictiveness, \(d_m\).

\[14\text{If platform } k \text{ obtains attention } a_k^i \text{ from each consumer } i, \text{ then } k\text{’s profit is } \int_{i \in [0,1]} a_k^i.\]
2. The consumer observes \((d_m)_{m \in M}\), then chooses the set \(M' \subset M\) of platforms to join. Namely, the consumer chooses \(M'\) to maximize the indirect utility calculated based on \(\sum_{m \in M'} \sum_{k \in P_m} u(a_k, s d_m)\).

3. After joining platforms, the consumer allocates attention across \(|\cup_{m \in M'} P_m|\) services, given the true addictiveness, \((d_m)_{m \in M'}\). The consumer’s payoff is her utility from \(\sum_{m \in M'} \sum_{k \in P_m} u(a_k, d_m)\) given the optimal allocation of attention. The payoff of each platform \(m\) is \(\sum_{k \in P_m} a_k\) if \(m \in M'\), and zero if \(m \notin M'\).

Point 1 implies that each platform \(m \in M\) chooses a single level of addictiveness for all services in \(P_m\). Point 2 implies that services operated by the same platform are tied—i.e., the consumer cannot join a strict nonempty subset of services in \(P_m\).

A model of price competition for the post-merger game is similarly defined: At the beginning of the game, each platform \(m \in M\) simultaneously sets addictiveness \(d_m\) and a price \(p_m\) for its services. If the consumer pays \(p_m\), she can access all of the services in \(P_m\), whose gross utility is \(\sum_{k \in P_m} u(a_k, d_m)\). The payoff of platform \(m\) is \(p_m\) or 0 if the consumer does or does not join it, respectively.

We consider the following two types of mergers.

**Definition 1.** A **symmetric merger** refers to \(M\) such that \(|P_m| = |P_\ell| \geq 2\) for all \(m, \ell \in M\) and \(M \geq 2\). An **all-but-one merger** refers to \(M\) such that \(M = \{K \{k\}, \{k\}\}\) for some \(k \in K\).

All-but-one merger is unique up to the identity of the non-merged platform. In contrast, there can be multiple symmetric mergers. The following result presents the welfare impacts of a merger (see Appendix E).

**Proposition 4.** Consider a model of competition for attention. A symmetric merger increases the addictiveness of all of the \(K\) services. An all-but-one merger increases the addictiveness of \(K - 1\) merged services and decreases that of the non-merged platform. In either case, a merger strictly decreases consumer surplus and total surplus.

To see the intuition, consider a symmetric merger such that four platforms merge into two platforms, each of which operates two services. After the merger, refusing to join one platform means that the consumer cannot use two services. Thus the consumer’s loss of not joining a
platform is larger in the post-merger game than in the original game. Each platform can then set higher addictiveness without deterring consumer participation. As a result, the consumer faces services with higher addictiveness after the merger. In the case of an all-but-one merger, a non-merged platform decreases the addictiveness of its service, responding to the higher addictiveness of the merged-platform. However, the consumer’s equilibrium payoff is equal to the one from the merged platforms, because she is indifferent between joining and not joining the non-merged platform. Thus, consumer surplus and total surplus decrease after the all-but-one merger.

Suppose now that platforms derive revenue exclusively from charging consumers for their services. In this case, a merger leads to higher monetary prices and a lower consumer surplus, but it does not distort service quality. Thus, we obtain the following result.

**Corollary 1.** Consider a symmetric merger or an all-but-one merger. The merger strictly decreases total surplus under competition for attention, but not when platforms compete on prices.

**Remark 2.** Our results (e.g., the efficient equilibrium under price competition) partly depend on the assumption that the consumer’s values for the services are deterministic. If the consumer had private values, price competition could lead to inefficiently high prices. In such a case, the comparison between attention competition and price competition would depend on the standard price distortion and the negative impact through high addictiveness under attention competition.

## 7 Discussion

### 7.1 Heterogeneity in Consumer Naivete

So far, we have considered a single consumer, or equivalently, multiple identical consumers. We now discuss a model of heterogeneous consumers. Specifically, suppose that a fraction \( \phi \in (0, 1] \) of consumers are fully sophisticated and have \( s = 1 \). The remaining fraction \( 1 - \phi \) of consumers are naive and have \( s = 0 \). Thus, they choose which platforms to join, believing that all platforms set \( d = 0 \). After joining platforms, all consumers allocate their attention given the true addictiveness. To ensure the existence of an equilibrium, we assume that there is an upper bound on feasible
levels of addictiveness, denoted by $\overline{d} > 0$.

If a fraction $\phi$ of sophisticated consumers is sufficiently large, platforms set the same addictiveness as in the benchmark of $\phi = 1$. The reason is as follows: The benchmark solution $d^*$ satisfies $K u \left( \frac{A}{K}, d^* \right) = (K - 1) u \left( \frac{A}{K-1}, d^* \right)$. If a platform deviates and increases its addictiveness, it can obtain more attention from naive consumers, but it discontinuously loses all sophisticated consumers. Formally, the gain for a platform from unilaterally increasing the addictiveness is at most $\phi \cdot \frac{(K-1) A}{K}$, whereas the loss is $(1 - \phi) \frac{A}{K}$. If $\phi > 1 - \frac{1}{K}$, the loss of losing sophisticated consumers exceeds the benefit of obtaining attention from naive consumers. As a result, the equilibrium is equal to the one in Proposition 1 with $s = 1$. For example, if there are two platforms and the majority of the consumers are sophisticated, the existence of naive consumers does not change the equilibrium.

If $\phi$ is close to 0 and $\overline{d}$ is large, there can be an equilibrium in which all platforms set the maximum addictiveness, and sophisticated consumers are excluded from the market. To see this, suppose that $\overline{d}$ satisfies $u(A, \overline{d}) \leq 0$, i.e., $\overline{d}$ is so large that sophisticated consumers will not join a platform with addictiveness $\overline{d}$. Suppose platforms 2, ..., $K$ choose $\overline{d}$. Platform 1 can either (i) choose $d = \overline{d}$ and obtain attention of $A/K$ from naive consumers, or (ii) choose a sufficiently low $d$ so that sophisticated consumers join only platform 1, but naive consumers allocate little attention to platform 1. If $\phi$ is close to 0, (i) is more profitable than (ii) for platform 1. Therefore, it is an equilibrium that all platforms set $\overline{d}$.

Generally, we can consider any $\phi \in (0, 1]$, or $s$ distributed across a population of consumers. We leave these cases for the future research.

### 7.2 Monopoly

So far, we have considered a market with multiple platforms, in which case there is a unique equilibrium. In contrast, a monopoly platform is indifferent between any addictiveness $d$ such that $u(A, d) \geq 0$, because any such $d$ ensures that the consumer joins the platform and allocates all of her attention. In particular, the monopoly platform may choose lower addictiveness than competing

15If all consumers are naive and the marginal utility goes to infinity as $d \to \infty$, then platforms will want to set $d = \infty$ to obtain the attention of naive consumers; however, the consumer problem under $d = \infty$ is not well-defined. Alternatively, we can assume that as $d \to \infty$, $u(\cdot, d)$ converges to some function that takes finite values and has a finite slope.
platforms. However, the attention monopoly may be empirically less relevant, because the “market for consumer attention” is broad. For example, even if Facebook becomes a monopoly in the social media market, it may still need to compete with other services, such as those provided by Google and Netflix. Therefore, the model with multiple platforms seems to be a better description of attention economy.

8 Conclusion

We study how firms compete for consumer attention by choosing the addictiveness of their services. We model a more addictive platform as a service that offers a lower utility and a higher marginal utility of allocating attention. There are two main findings. Overall, the equilibrium addictiveness is inefficiently high, and a less competitive market leads to greater addictiveness. Our result captures a welfare loss due to market power that might not appear in the standard antitrust analysis based on monetary prices. Second, the impacts of policies and mergers relevant to digital addiction are likely to depend on business models and consumer sophistication. We hope that our work serve as a foundation for more studies to understand the interaction between competition and digital addiction.

References


Appendix

A Proof of Proposition 1

Proof. CLAIM 1: In the unique equilibrium, all platforms choose \( \frac{d_s(K)}{s} \).

Step 1: There is a unique \( d_s(K) \) that satisfies (3). To show this, define

\[
f(K, d) := K \cdot u \left( \frac{A}{K}, d \right) - (K - 1) \cdot u \left( \frac{A}{K-1}, d \right).
\]

The function \( f(K, d) \) is the difference between utilities when the consumer uses \( K \) platforms and when she uses \( K - 1 \) platforms, given optimally allocating attention. Define \( g(y, d) := y \cdot u \left( \frac{A}{y}, d \right) \).

Then \( g_1(y, d) = u \left( \frac{A}{y}, d \right) - \frac{A}{y} \cdot u_1 \left( \frac{A}{y}, d \right) \), which is strictly decreasing in \( d \) because \( \frac{\partial g_1}{\partial d} = u_2 - \frac{A}{y} u_{12} < 0 \) under \( u_2 < 0 \) and \( u_{12} > 0 \). As a result, \( f(K, d) = \int_{K-1}^{K} g_1(y, d) dy \) is strictly decreasing in \( d \). Also, \( f(K, 0) > 0 \), and \( f(K, d) < 0 \) if \( u \left( \frac{A}{K}, d \right) = 0 \) for any \( d > 0 \). Thus, there is a unique \( d_s(K) \) such that \( K \cdot u \left( \frac{A}{K}, d_s(K) \right) = (K - 1) \cdot u \left( \frac{A}{K-1}, d_s(K) \right) \).

Step 2: There is an equilibrium in which each platform sets \( d_s(K) = d_s(K)/s \). First, we show that given \( d_s(K) \), the consumer prefers to join all platforms. Define \( h(y) = \frac{A}{y} u \left( y, sd \right) \), \( y_K = \frac{A}{K} \), and \( y_{K-1} = \frac{A}{K-1} \). Then, \( d^* = d_s(K) \) satisfies \( \frac{A}{y_K} u(y_K, sd^*) = \frac{A}{y_{K-1}} u(y_{K-1}, sd^*) \). This equality implies that a straight line that goes through the origin and \( (y_K, Au(y_K, sd^*)) \) has the same slope as the one that goes through the origin and \( (y_{K-1}, Au(y_{K-1}, sd^*)) \) (see Figure A.1). Because \( Au(\cdot, sd^*) \) is concave, for any \( k \leq K - 2 \), it follows that \( \frac{A}{y_k} u(y_k, sd^*) < \frac{A}{y_{K-1}} u(y_{K-1}, sd^*) \). As a result, for each \( k \leq K - 1 \), the consumer prefers joining \( k \) platforms to joining any \( k - 1 \) platforms (given her perceived addictiveness).

Second, we show that no platform has a profitable deviation. Without loss of generality, we consider the incentive of platform 1. If it increases \( d_1 \), the consumer joins only platforms 2, \ldots, K.
to achieve the same payoff as without platform 1’s deviation. If platform 1 decreases $d_1$, the consumer still joins all platforms. To see this, for each $J \in \{1, \ldots, K\}$ and $d \geq 0$, let $V_J(d)$ denote the consumer’s indirect utility when she joins platforms $1, \ldots, J$, platform 1 chooses $d$, and other platforms choose $d^*$ defined above. By the envelope theorem, we have $V_J'(d) = u_2(a_1(J, d), s_d)$, where $a_1(J, d)$ is the attention allocated to platform 1 given $(J, d)$. Because $a_1(J, d)$ is decreasing in $J$, $V_J'(d)$ is decreasing in $J$. Thus, we have $V_K'(d) < V_J'(d)$ for all $J < K$. Also, we have $V_K(d^*) = V_{K-1}(d^*)$. As a result, if $d < d^*$, we have $V_K(d^*) > V_J(d^*)$ for all $J < K$, i.e., the consumer prefers to join all platforms. Platform 1 then receives a smaller attention because of the complementarity assumption ($u_{12} > 0$). Therefore, no platform has a profitable deviation from $d_s(K)$.

**Step 3:** The above equilibrium is a unique equilibrium. To show this, take any equilibrium. First, we show that all platform choose the same addictiveness in any equilibrium (i.e., any pure-strategy subgame perfect equilibrium). Suppose to the contrary that there is an equilibrium in which platforms choose $(d_k^*)_{k \in K}$ such that (without loss of generality) $d_2 = \max_k d_k^* > \min_k d_k^* = d_1$. In the equilibrium, for each $k \in K$, the consumer is indifferent between joining and not joining platform $k$. Indeed, if the consumer strictly prefers to join $k$, it can increase $d_k$ to obtain greater attention. Suppose now that platform 1 deviates and increases its addictiveness to $d_1 \in (d_1^*, d_2^*)$. Let $K'$ denote the set of platforms the consumer joins, following the deviation. If $1 \not\in K'$, then it is optimal for the consumer to choose $K' = \{2, \ldots, K\}$ (otherwise, she would not have joined some of the platforms $2, \ldots, K$ before platform 1’s deviation). Compared to such a choice, the consumer is strictly better off by replacing platform 2 with platform 1, because $d_1 < d_2^*$. Thus, we have $1 \in K'$. Then, the consumer allocates a greater attention to platform 1 after the deviation, because the consumer now joins fewer platforms and faces a higher marginal utility of allocating attention to platform 1. This is a contradiction. As a result, in any equilibrium, $d_k^*$ is the same for all $k \in K$.

Second, take any equilibrium in which all platforms choose the same addictiveness. If the consumer’s indifference condition (3) fails, then one of the following holds: (i) the left-hand side is strictly greater, in which case a platform prefers to deviate and increase its addictiveness, or (ii) the right-hand side is greater, in which case the consumer does not join at least one platform, say $k$. If (ii), platform $k$ can choose a sufficiently small $d_k$ to induce the consumer’s participation. As
a result, both (i) and (ii) lead to contradictions.

CLAIM 2: \( d_s(K) \) is strictly decreasing in \( K \), and \( \lim_{K \to \infty} d_s(K) = 0 \).

First, suppose to the contrary that \( d_s(K' + 1) \geq d_s(K') \) for some \( K' \geq 2 \). Consider the consumer’s choice between joining \( K' - 1 \) platforms \((1, \ldots, K' - 1)\) and \( K' \) platforms \((1, \ldots, K')\) when there are \( K' + 1 \) platforms, i.e., \( K = K' + 1 \). If there are \( K' \) platforms, the consumer is indifferent between joining platforms \( 1, \ldots, K' \) and \( 1, \ldots, K' - 1 \). Because platforms choose higher addictiveness under \( K = K' + 1 \), the consumer’s loss of not joining platform \( K' \) is smaller and the benefit of allocating remaining attention to the \( K' - 1 \) platforms is greater under \( K = K' + 1 \) than under \( K = K' \). Thus, the consumer weakly prefers joining \( K' - 1 \) platforms to \( K' \) platforms if \( K = K' + 1 \). This contradicts Step 2 above. Thus \( d_s(K) \) is strictly decreasing in \( K \).

Second, suppose to the contrary that \( \lim_{K \to \infty} d_s(K) = \hat{d} > 0 \). The consumer’s utility from each platform is \( u\left(\frac{A}{K}, d_s(K)\right)\), which converges to \( u(0, \hat{d}) < 0 \). This is a contradiction. Hence, \( \lim_{K \to \infty} d_s(K) = 0 \).

CLAIM 3: Consumer surplus and total surplus are strictly increasing in \( K \) and \( s \). Because \( d_s(K) \) is decreasing in \( K \) and \( s \), consumer surplus increases in \((K, s)\). Total surplus is the sum of consumer surplus and the total payoff of all platforms. As the total payoff to all platforms are constant at \( A \), the total surplus also increases with \((K, s)\). □

![Figure A.1: The consumer prefers to use all platforms.](image-url)
B Proof of Proposition 2

Proof. Claim 1: There is an equilibrium in which each platform $k$ sets $d_k = 0$ and $p_k = p^*$. 

Suppose each platform $k$ sets $(d_k, p_k) = (0, p^*)$. First, we show that the consumer prefers to join all platforms (the argument is the same as Proposition 1). Define $h(y) = \frac{A}{y_k} [u(y, 0) - p^*]$, $y_K = \frac{A}{K}$, and $y_{K-1} = \frac{A}{K-1}$. Then, we have $\frac{A}{y_k} [u(y_K, 0) - p^*] = \frac{A}{y_{K-1}} [u(y_{K-1}, 0) - p^*]$. This equality implies that a straight line that goes through the origin and $(y_K, A[u(y_K, 0) - p^*])$ has the same slope as the one that goes through the origin and $(y_{K-1}, A[u(y_{K-1}, 0) - p^*])$. Because $Au(\cdot, 0) - p^*$ is concave, for any $k \leq K - 2$, it follows that $\frac{A}{y_k} [u(y_k, 0) - p^*] < \frac{A}{y_{K-1}} [u(y_{K-1}, 0) - p^*]$. As a result, for each $k \leq K - 1$, the consumer prefers joining $k$ platforms to joining any $k-1$ platforms (given her perceived addictiveness).

Suppose platform $k$ deviates and chooses $(d_k^*, p_k^*)$. If $d_k^* > 0$, platform $k$ has to set $p_k^* < p^*$; otherwise, the consumer strictly prefers to join only platforms $2, \ldots, K$. In this case, the deviation reduces $k$’s payoff. Conditional on $d_k^* = 0$, $p^*$ is the maximum price that platform $k$ can charge, because the consumer is indifferent between joining $K - 1$ and $K$ platforms at price $p^*$. Thus, platform $k$ has no profitable deviation.

Claim 2: The above equilibrium is a unique equilibrium.

Take any equilibrium, and suppose each platform $k$ chooses $(d_k^*, p_k^*)$. First, we show that the consumer joins all platforms in equilibrium. Fix $\hat{k} \in K$, and suppose platform $\hat{k}$ sets $(d_{\hat{k}}, p_{\hat{k}}) = (0, 0)$, which may or may not be a deviation. Let $K_0$ denote the set of platforms the consumer joins, following $(d_k, p_k) = (0, 0)$. Take any $K' \subset K$ such that $\hat{k} \not\in K'$. First, if $d_j^* > 0$ for some $j \in K'$, then the consumer strictly prefers joining $(K' \setminus \{j\}) \cup \{\hat{k}\}$ to joining $K'$. Second, if $d_j^* = 0$ for all $j \in K'$ or $K' = \emptyset$, then the consumer strictly prefers $K' \cup \{\hat{k}\}$ to $K'$. Thus, for any set $K'$ of platforms such that $\hat{k} \not\in K'$, we can find some set $S$ of platforms such that $\hat{k} \in S$ and the consumer strictly prefers $S$ to $K'$. As a result, for a sufficiently small $p_{\hat{k}} > 0$ and $d_{\hat{k}} = 0$, the consumer still joins platform $\hat{k}$. This argument implies that any platform earns a positive profit in any equilibrium. Therefore, the consumer joins all platforms.

Second, we show all platforms set zero addictiveness in any equilibrium. Suppose to the contrary that $d_k^* > 0$ for some $k$. Suppose platform $k$ deviates and chooses $(d_k, p_k) = (0, p_k^*)$. Before the deviation, the consumer weakly prefers joining all platforms to joining any set $K'$ of platforms.
that does not contain $k$. Thus, after the deviation to $(0, p_k^*)$, the consumer strictly prefers to joining platform $k$. As a result, platform $k$ can slightly increase its price while retaining the consumer. This is a contradiction.

We have shown that in any equilibrium, the consumer joins all platforms, which set zero addictiveness. The price of each platform makes the consumer indifferent between joining and not joining the platform; otherwise, the platform can deviate by slightly increasing its price. Therefore, $(d_k^*, p_k^*) = (0, p^*)$ is a unique equilibrium.

**Claim 3:** There is a $s^* \in (0, 1)$ such that the consumer is better off under price competition if and only if $s \leq s^*$.

First, we show that the consumer with $s = 1$ is strictly better off under competition for attention, in which the consumer’s payoff is $K u\left(\frac{A}{K}, d(K)\right)$. Under price competition, her payoff is

\[
K u\left(\frac{A}{K}, 0\right) - K \left[ Ku\left(\frac{A}{K}, 0\right) - (K-1)u\left(\frac{A}{K-1}, 0\right)\right]
\]

\[
= K(K-1) \left[ u\left(\frac{A}{K-1}, 0\right) - u\left(\frac{A}{K}, 0\right)\right]
\]

\[
< K(K-1) \left[ u\left(\frac{A}{K-1}, d(K)\right) - u\left(\frac{A}{K}, d(K)\right)\right]
\]

\[
= Ku\left(\frac{A}{K}, d(K)\right).
\]

The inequality holds because $u_{12} > 0$. The last equality is from $K u\left(\frac{A}{K}, d(K)\right) = (K-1)u\left(\frac{A}{K-1}, d(K)\right)$. Therefore, the consumer is strictly better off when platforms compete only on addictiveness.

Second, we prove there is a unique $s^*$ that has the desired properties. Take any $s \in (0, 1]$. Under attention competition, the consumer’s equilibrium payoff $K u\left(\frac{A}{K}, \frac{d(K)}{s}\right)$ is continuous and strictly increasing in $s$. For a small $s$ such that $u(A/K, d) < 0$, the consumer obtains a negative payoff, in which case she is strictly better off under price competition. The intermediate value theorem implies there is a unique $s^*$ such that the claim is established. \qed
C Proof of Lemma 1

Proof. Define
\[ f(A, d) := K \cdot u\left(\frac{A}{K}, d\right) - (K - 1) \cdot u\left(\frac{A}{K-1}, d\right) = 0. \]

Let \( d(A) \) denote the equilibrium addictiveness in Proposition 1. Applying the implicit function theorem to \( f(A, d(A)) = 0 \), we have
\[ d'(A) = -\frac{f_A}{f_d} = -\frac{u_1\left(\frac{A}{K}, d(A)\right) - u_1\left(\frac{A}{K-1}, d(A)\right)}{K \cdot u_2\left(\frac{A}{K}, d(A)\right) - (K - 1) \cdot u_2\left(\frac{A}{K-1}, d(A)\right)}. \]  

(A.1)

The numerator \( u_1\left(\frac{A}{K}, d(A)\right) - u_1\left(\frac{A}{K-1}, d(A)\right) \) is positive because \( u \) is strictly concave in \( a \), and \( \frac{A}{K} < \frac{A}{K-1} \) for all \( K \geq 2 \). The denominator is negative:
\[ K \cdot u_2\left(\frac{A}{K}, d(A)\right) - (K - 1) \cdot u_2\left(\frac{A}{K-1}, d(A)\right) \]
\[ = K \cdot \left[ u_2\left(\frac{A}{K}, d(A)\right) - u_2\left(\frac{A}{K-1}, d(A)\right) \right] + u_2\left(\frac{A}{K-1}, d(A)\right) < 0. \]  

(A.2)

Thus, we obtain \( d'(A) > 0. \)

D Proof of Proposition 3

Proof. First, we show Point 1. Let \( d(A) \) denote the equilibrium addictiveness when the consumer is fully sophisticated. For a given \( s \), the equilibrium addictiveness is \( d_s(A) = \frac{d(A)}{s} \). The equilibrium payoff is \( U_s(A) = Ku\left(\frac{A}{K}, d_s(A)\right) \). Using (A.1), we have
\[ U'_s(A) = u_1\left(\frac{A}{K}, d_s(A)\right) + K u_2\left(\frac{A}{K}, d_s(A)\right) \cdot \frac{d'(A)}{s} = u_1\left(\frac{A}{K}, d_s(A)\right) + K u_2\left(\frac{A}{K}, d_s(A)\right) \cdot \left( -\frac{u_1\left(\frac{A}{K}, d(A)\right) - u_1\left(\frac{A}{K-1}, d(A)\right)}{K \cdot u_2\left(\frac{A}{K}, d(A)\right) - (K - 1) \cdot u_2\left(\frac{A}{K-1}, d(A)\right)} \right) \cdot \frac{1}{s}. \]  

(A.3)
Given (A.2), we can rewrite $U_s'(A) > 0$ as

$$s \cdot u_1\left(\frac{A}{K}, \frac{d(A)}{s}\right) \cdot \left(K \cdot u_2\left(\frac{A}{K}, d(A)\right) - (K - 1) \cdot u_2\left(\frac{A}{K - 1}, d(A)\right)\right) < K \cdot u_2\left(\frac{A}{K}, d(A)\right) \cdot \left(u_1\left(\frac{A}{K}, d(A)\right) - u_1\left(\frac{A}{K - 1}, d(A)\right)\right).$$

The inequality is written as

$$\left((s + d(A)) \cdot w'\left(\frac{A}{K}\right) \cdot \left(K \cdot u_2\left(\frac{A}{K}, d(A)\right) - (K - 1) \cdot u_2\left(\frac{A}{K - 1}, d(A)\right)\right)\right) \lesssim\!
\begin{array}{c}
\begin{cases}
0 & \text{if } s > s^* \\
<0 & \text{if } s < s^*
\end{cases}
\end{array}
\right)
< K \left[w\left(\frac{A}{K}\right) - c\right] \cdot \left(u_1\left(\frac{A}{K}, d(A)\right) - u_1\left(\frac{A}{K - 1}, d(A)\right)\right) \cdot \left(u_1\left(\frac{A}{K}, d(A)\right) - u_1\left(\frac{A}{K - 1}, d(A)\right)\right).$$

The left-hand side is decreasing in $s$ and the right-hand side is independent of $s$. Therefore, there is a cutoff $s^*$ such that the inequality holds if and only if $s > s^*$. The cutoff $s^*$ could be 0 or 1.

Second, we show Point 2. Because $s = 1$, the consumer’s equilibrium payoff is $U(A) := K u\left(\frac{A}{K}, d(A)\right)$. Plugging $s = 1$ into (A.3), and rearranging it, we can write $U'(A) > 0$ as

$$u_1\left(\frac{A}{K}, d(A)\right) \cdot \left(K \cdot u_2\left(\frac{A}{K}, d(A)\right) - (K - 1) \cdot u_2\left(\frac{A}{K - 1}, d(A)\right)\right) < K u_2\left(\frac{A}{K}, d(A)\right) \cdot \left(u_1\left(\frac{A}{K}, d(A)\right) - u_1\left(\frac{A}{K - 1}, d(A)\right)\right),$$

which reduces to

$$K u_2\left(\frac{A}{K}, d(A)\right) \cdot u_1\left(\frac{A}{K - 1}, d(A)\right) < (K - 1) u_1\left(\frac{A}{K}, d(A)\right) \cdot u_2\left(\frac{A}{K - 1}, d(A)\right).$$

Given $u_1(a, d) = \frac{1 + a}{1 + a}$ and $u_2(a, d) = \log(1 + a) - c$, the inequality becomes

$$K \cdot \left(\log\left(1 + \frac{A}{K}\right) - c\right) \cdot \frac{1 + d(A)}{1 + \frac{A}{K - 1}} < (K - 1) \cdot \frac{1 + d(A)}{1 + \frac{A}{K}} \cdot \left(\log\left(1 + \frac{A}{K - 1}\right) - c\right).$$
which is equivalent to
\[ K \left( 1 + \frac{A}{K} \right) \left( \log \left( 1 + \frac{A}{K} \right) - c \right) < (K - 1) \left( 1 + \frac{A}{K - 1} \right) \left( \log \left( 1 + \frac{A}{K - 1} \right) - c \right). \]

To show this inequality, we show that \( g(x) = x \left( 1 + \frac{A}{x} \right) \left( \log \left( 1 + \frac{A}{x} \right) - c \right) \) is decreasing on \( x \in [1, \infty) \). It holds that
\[
g'(x) = \left( 1 + \frac{A}{x} \right) \left( \log \left( 1 + \frac{A}{x} \right) - c \right) - \frac{A}{x} \left( \log \left( 1 + \frac{A}{x} \right) - c \right) - \frac{A}{x} = \log \left( 1 + \frac{A}{x} \right) - c - \frac{A}{x} < 0.
\]

The last inequality holds because of \( u_2(A, d) < 0 \), which implies \( \log \left( 1 + \frac{A}{x} \right) - c < 0 \) for all \( x \geq 1 \). Therefore \( U'(A) > 0 \), i.e., reducing \( A \) harms a sophisticated consumer.

\[ \square \]

E Proof of Proposition 4

Proof. First, we consider a symmetric merger. Suppose that there are originally \( K := L \cdot M \) platforms, and they merge and become \( M \) platforms. Each platform operates \( L \) services. Suppose \( s = 1 \). After the merger, the unique equilibrium addictiveness \( d^*_{M} \) solves
\[ LM \cdot u \left( \frac{A}{LM}, d^*_{M} \right) = (M - 1)L \cdot u \left( \frac{A}{(M - 1)L}, d^*_{M} \right). \]

Compare \( d^*_{M} \) to the one in the pre-merger equilibrium, in which the addictiveness \( d^* \) satisfies
\[ LM \cdot u \left( \frac{A}{LM}, d^* \right) = (LM - 1) \cdot u \left( \frac{A}{LM - 1}, d^* \right). \]

Recall that \( y \cdot u \left( \frac{A}{y}, d \right) \) is increasing in \( y \); thus,
\[ LM \cdot u \left( \frac{A}{LM}, d \right) - (M - 1)L \cdot u \left( \frac{A}{(M - 1)L}, d \right) > LM \cdot u \left( \frac{A}{LM}, d \right) - (LM - 1) \cdot u \left( \frac{A}{LM - 1}, d \right). \]

Also, \( \frac{\partial^2 \}{\partial d \partial y} \left( yu \left( \frac{A}{y}, d \right) \right) < 0 \). Thus, the left- and right-hand sides of the above inequality are decreasing in \( d \). As a result, \( d \) that solves \( LM \cdot u \left( \frac{A}{LM}, d \right) - (M - 1)L \cdot u \left( \frac{A}{(M - 1)L}, d \right) = 0 \) is strictly greater than \( d \) that solves \( LM \cdot u \left( \frac{A}{LM}, d \right) - (LM - 1) \cdot u \left( \frac{A}{LM - 1}, d \right) = 0 \). Thus, \( d^*_{M} > d^* \).

The consumer uses the same set of services in equilibrium both in the pre-merger and post-merger
games. Thus, the consumer payoff and total surplus are lower in the post-merger game. For $s < 1$, the same argument applies to $\frac{d_M^*}{s}$ and $\frac{d^*}{s}$.

Second, we consider an all-but-one merger. Without loss of generality, suppose platforms 2, \ldots, $K$ merge and become platform $M$. Let $d_1$ and $d_M$ denote their addictiveness, with the equilibrium values being $d_1^*$ and $d_M^*$. In equilibrium the consumer joins all platforms, because any platform can choose $d_k = 0$ to obtain a positive amount of attention. Let $a_1(d_1, d_M)$ and $A_M(d_1, d_M)$ denote the attention allocated to platform 1 and $M$, respectively, given $(d_1, d_M)$. In equilibrium, we have

$$u(a_1(d_1^*, d_M^*), d_1^*) + (K - 1)u\left(\frac{A_M(d_1^*, d_M^*)}{K - 1}, d_M\right)$$  \hspace{1cm} (A.4)

$$= u(A, d_1^*)$$  \hspace{1cm} (A.5)

$$= (K - 1)u\left(\frac{A}{K - 1}, d_M^*\right).$$  \hspace{1cm} (A.6)

The expression (A.4) is the consumer’s payoff of joining both platforms 1 and $M$. Platform $M$ consists of $K - 1$ symmetric services with decreasing marginal utilities, so the consumer allocates $\frac{A_M(d_1^*, d_M^*)}{K - 1}$ to each of the $K - 1$ services. The expressions (A.5) and (A.6) are the consumer’s payoffs of joining only platform 1 and $M$, respectively. If any of the above equalities fails, some platform will have a profitable deviation. The rest of the proof consists of three steps.

**Claim 1:** The merger of $K - 1$ platforms increases the addictiveness of the merged services and decreases that of the non-merged platform.

Let $d_0$ denote the equilibrium addictiveness before the merger. First, we show $d_M^* > d_0$. Recall that given $d_0$, the consumer is indifferent between joining $K$ and $K - 1$ platforms, and she strictly prefers joining $K$ platforms to joining $k \leq K - 2$ platforms. As a result, we have

$$u(a_1(d_0, d_0), d_0) + (K - 1)u\left(\frac{A_M(d_0, d_0)}{K - 1}, d_0\right) > u(A, d_0)$$

$$\Rightarrow f(d_0) := u(a_1(d_0, d_M^*), d_0) + (K - 1)u\left(\frac{A_M(d_0, d_M^*)}{K - 1}, d_M^*\right) - u(A, d_0) > 0.$$
By the envelope theorem,

\[ f'(d_0) = u_2(a_1(d_0, d_M), d_0) - u_2(A, d_0) < 0, \]

because \( u_{12} > 0 \) and \( a_1(d_0, d_M) < A \). Note that (A.4) is equal to (A.5). Also, we have \( f(d_0) > 0 \) and \( f'(d_0) < 0 \). As a result, \( d_1^* > d_0 \) holds. However, this is a contradiction: If \( d_1^* > d_0 \), then platform \( M \) can choose \( d_M = d_0 \) to obtain all attention \( A \). Here, we use the fact that platform \( M \) consists of \( K - 1 \) platforms and that the consumer is indifferent between joining \( K \) platforms and \( K - 1 \) platforms when all platforms choose \( d_0 \). Therefore, \( d_M^* > d_0 \).

Next, we show platform 1 reduces its addictiveness after the merger, i.e., \( d_1^* < d_0 \). Note that \( d_M^* > d_0 \) implies

\[ u(a_1(d_0, d_M^*), d_0) + (n - 1)u \left( \frac{A_M(d_0, d_M^*)}{K - 1}, d_M^* \right) < (K - 1)u \left( \frac{A}{K - 1}, d_M^* \right), \]  

(A.7)

because (A.4) equals (A.6). For the left-hand side to be equal to the right-hand side, \( d_1^* < d_0 \) must hold.

CLAIM 2: There is a pure-strategy subgame perfect equilibrium in the post-merger game.

First, let \( \bar{d} \) be the unique value that satisfies \( u(A, \bar{d}) = 0 \). Consider the equation

\[ u(a_1(d_1, d_M), d_1) + (K - 1)u \left( \frac{A_M(d_1, d_M)}{K - 1}, d_M \right) = u(A, d_1). \]  

(A.8)

Fix \( d_1 \in [0, \bar{d}] \). If \( d_M = 0 \), the left-hand side is weakly greater. As \( d_M \to \infty \), the left-hand side goes to \(-\infty\). Also, the left-hand side is continuous and strictly decreasing in \( d_M \). As a result, there is a unique \( d_M \) (denote by \( d_M(d_1) \)) that satisfies the above equation. \( d_M(d_1) \) is continuous. To show \( d_M(d_1) \) is decreasing, define

\[ g(d_1, d_M) = u(A, d_1) - u(a_1(d_1, d_M), d_1) - (K - 1)u \left( \frac{A_M(d_1, d_M)}{K - 1}, d_M \right). \]  

(A.9)

Note that \( d_M(d_1) \) solves \( g(d_1, d_M) = 0 \) with respect to \( d_M \). By the envelope theorem,

\[ g_1(d_1, d_M) = u_2(A, d_1) - u_2(a_1(d_1, d_M), d_1) \geq 0, \]
because \( u_{12}(a, d) > 0 \). Because \( g_2(u_1, d_M) > 0 \), for the equation \( g(d_1, d_M) = 0 \) to hold, \( d_M \) must decrease whenever \( d_1 \) increases. As a result, \( d_M(d_1) \) is weakly decreasing.

Next, for each \( d_1 \in [0, \bar{d}] \), let \( \hat{d}_M(d_1) \) solve

\[
(K - 1)u\left(\frac{A}{K - 1}, d_M\right) = u(A, d_1). \tag{A.10}
\]

By the similar argument as above, we can show that \( \hat{d}_M(d_1) \) is unique, continuous, and strictly increasing. At \( d_1 = 0 \), the left-hand side of (A.8) is weakly greater than that of (A.10). Thus, \( d_M(0) \geq \hat{d}_M(0) \). At \( d_1 = \bar{d} \), the left-hand side of (A.8) is weakly smaller than that of (A.10), because

\[
u(a_1(d_1, d_M), d_1) + (K - 1)u\left(\frac{A_M(d_1, d_M)}{K - 1}, d_M\right) \leq (K - 1)u\left(\frac{A_M(d_1, d_M)}{K - 1}, d_M\right) \leq (K - 1)u\left(\frac{A}{K - 1}, d_M\right).
\]

Here, the first inequality is from \( u(a_1(d_1, d_M), d_1) \leq 0 \). Thus, \( d_M(\bar{d}) \leq \hat{d}_M(\bar{d}) \). Because \( d_M(d_1) \) is weakly decreasing and \( \hat{d}_M(d_1) \) is strictly increasing, they have a unique crossing point, which corresponds to the unique equilibrium.

Claim 3: The merger decreases consumer surplus and total surplus.

The consumer surplus in the post-merger game is \((K - 1)u\left(\frac{A}{K - 1}, d_M^*\right)\), and the one in the pre-merger game is \((K - 1)u\left(\frac{A}{K - 1}, d_0\right)\). Because \( d_M^* > d_0 \), the consumer is worse off after the merger. The merger does not affect the allocation of attention, and thus the payoff of each platform remains the same. As a result, the merger decreases total surplus. \(\square\)