Informational Bundling, Freemium, and Content Menu Design*

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Abstract

We study a monopoly platform's optimal screening contracts when the platform designs a menu of contracts such that heterogeneous consumers self-selectively choose a subset of a continuum of vertically differentiated content. We show that the platform optimally excludes some high-quality content from low-type consumers when her business model is advertising revenue from consumers' attention, while such a scheme is never optimal when the platform adopts a subscription-based business model. When a negative price cannot subsidize content consumption, the platform may optimally offer a *freemium contract* in which some consumers consume a subset of content for free of charge but are exposed to ads. Lastly, the platform may have a socially excessive incentive to show advertising to low-type consumers to reduce the information rent yielded to high-type consumers if ads become a greater nuisance to the high-type consumers. Our paper shows that *informational bundling* arises only when the platform adopts an ad-funded business model, not a subscription-based one. Advertising with no subscription fee allows the platform to induce consumers to accept the content allocation of which the consumers would not consume each and every content when the individual quality was known ex-ante. (*JEL codes*: D4, D82, L5, M3)

Keywords: screening, advertising, media platform, freemium, informational bundling

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1 Introduction

Different media platforms use a variety of menu contracts which differ in accessible content, amount of ads, prices, etc. Many platforms have adopted the so-called *freemium* business model, of which the basic free versions involve advertising. However, all freemium models are not the same. For instance, YouTube and Spotify put no restriction on the set of accessible content regardless of tiers, though the premium versions involve no ads (with better functionalities) for a subscription fee. By contrast, in the case of newspapers, both subscribers and non-subscribers are subject to advertising while they limit content accessible to non-subscribers. *New York Times* and the *Washington Post* limit the number of free articles that non-subscribers can access per month, whereas newspapers such as *Le Monde* and the *Figaro* offer the most prestigious content only to their paid subscribers.

The media market is enormous: an average consumer spends 495 minutes a day consuming media.[1] Business models of media platforms are significantly affecting consumer welfare and firm profits. Yet, theoretical developments regarding their business models have been lacking. This paper is one attempt to fill the gap. We consider a monopoly platform providing a continuum of vertically differentiated content and study the design of optimal screening contracts. We enrich a well-established monopolistic screening framework (Mussa and Rosen, 1978; Maskin and Riley, 1984) by allowing the platform to decide not only the amount of content but also the average quality. In addition, we allow the platform to choose whether or not content consumption is subject to advertising. This rich framework allows us to address several interesting questions. How does advertising affect the content allocation design of the platform and consumer surplus? When is the freemium strategy optimal, and how does it affect content allocation and consumer surplus? How do type-dependent nuisance costs affect content allocation and consumer surplus?

We consider a monopolist media platform that provides content to consumers and can intermediate advertisers and consumers.[2] There is a unit mass of consumers who have heterogeneous types in terms of their taste for quality: we consider both a model of continuous types and two types (i.e., high and low types). Consumers incur a certain attention cost to consume digital content as an opportunity cost. We use the term *content allocation* to refer

[1] Source: https://www.zenithmedia.com/consumers-will-spend-800-hours-using-mobile-internet-devices-this-year/

[2] For example, see Choi, Jeon, and Kim (2015), Sato (2019), Lin (2020), Carroni and Paolini (2020), and Jeon, Kim, and Menicucci (2021).

to the set of content consumed by a consumer. *Top-down* allocations trim off some content below a threshold quality. By contrast, *bottom-up* allocations mean that the non-subscribing users cannot access some top-quality content.

We first consider the subscription-based business model without advertising as a benchmark. The first-best subscription-based contract entails top-down allocations for all types of consumers. The set of content rendered to the higher type of consumers is larger than that of the lower type of consumers, which means that some content is exclusively offered to the higher type of consumers who pay more. With private information regarding the consumer type, the platform introduces a downward distortion (except the highest type) by restricting the set of accessible content with respect to the first-best. As usual, the trade-off between efficiency and rent extraction makes θ-consumer's *virtual valuation*, denoted by θ^v, smaller than their true valuation θ (except the highest type), in the first-best contract.[3] The platform's optimal second-best contract is to offer the content of quality q as far as a consumer's virtual utility of consuming that content net of her attention cost (a) is non-negative, that is, $\theta^v q - a \geq 0$. When the virtual valuation is small enough or negative, it excludes those types from consumption. Therefore, both the first-best and the second-best contracts entail top-down allocations and informational bundling does not play any role in both contracts.

Then we introduce advertising in a simple manner. It generates ad revenues (r) to the platform but a nuisance cost (c) to consumers, which is assumed to be multiplicative to the quality of content. The higher the quality, the larger the ad nuisance. We analyze the optimal ad-funded contract in a progressive way. In the beginning, we assume that the nuisance cost does not depend on the consumer type and that the platform can subsidize content consumption with a negative price. Then we introduce a non-negative price constraint as subsidizing consumption is subject to ex post moral hazard of consumers.[4] Last, we consider type-dependent nuisance costs and consider the case in which high types suffer larger nuisance costs than low types.

The first-best contract with advertising again leads to top-down allocations.[5] However, the advertising generates interesting departures in the case of the second-best contract.

[3]The trade-off between rent extraction and efficiency is well known. See Laffont and Martimort (2002) for details.

[4]Recently, non-negative price constraint has been paid attention to in the economics of two-sided platforms, e.g., see Choi and Jeon (2021, 2022) and therein references.

[5]If the marginal ad revenue exceeds the attention cost, then the platform finds no reason to trim off any content and thus will offer *full* allocation with the entire set of content. See more details in Section 4.1.

Specifically, θ-type consumer's content allocation now depends on the following two terms: $\theta^v - c$, the term capturing a net profit per quality, and $r - a$, the term capturing a net profit from one unit of content served. In this revised environment, even the relationship of $\theta^v q - a > 0$ no longer warrants $(\theta^v - c)q + (r - a) > 0$ because either $\theta^v - c$ or $r - a$ can take on a negative value. Remarkably, if ads generate a large revenue ($r - a > 0$) but impose high nuisance costs relative to the virtual valuation ($\theta^v - c < 0$), then the platform finds it better to apply bottom-up allocations. This is because the platform wants to increase the measure of content for ad revenue but with the lowest average quality of the package to minimize the information rent given to higher-type consumers. Such a move is never optimal without advertising.

One novel concept we introduce in our paper is *informational bundling*. Suppose a consumer is willing to consume the whole content even when she does not observe each quality. Now imagine that she can observe individual quality. If she keeps consuming all the content as before, we say that informational bundling did not arise as the consumer consumes the same set of content even under perfect information. But if she does not consume a subset of content, then we say this subset of content is informational bundled with the complementary subset.[6] One main result is that informational bundling arises only when the platform uses ad-funded business models but does not occur under the subscription-based business model. Therefore it plays an important role when evaluating the welfare consequences of the ad-funded business model relative to the subscription-based model.

We then consider the so-called *non-negative price constraint* under which the platform cannot use a negative price to subsidize content consumption. Then, when the non-negative price constraint binds, the optimal ad-funded contract entails the freemium contract: the platform offers some content for free with advertising. The low-type consumers are offered either top-down allocations (like the *New York Times*) or bottom-up allocations (like the *Figaro*) but no explicit price. In contrast, the high-type consumers have full access to the entire content with a positive subscription fee (and possibly with no ads). Therefore, the prevalent freemium business strategy naturally arises in optimum through a simple screening mechanism. Relative to the situation where the platform is able to subsidize content consumption, the binding non-negative price constraint expands (reduces) the content set and total quality

[6] An example of informational bundling can be the news homepage of Daum, a major Korean news portal. Daum shows only the title of each news article (or a part of the title if the title is long) without any information about the newspaper's identity that produced the article, without any other snippet, and without pictures. This is in contrast with the practice of Google News, which provides the full title with the news source and a picture.

consumed by low types under bottom-up allocations (top-down allocations), which implies higher (lower) consumer surplus.

Finally, when ad nuisance cost is higher for high types than low types, exposing low types to advertising enables the platform to extract information rent from high types, which lowers consumer surplus. Therefore, the platform may introduce advertising for rent extraction even if advertising technology is socially inefficient. However, high types' suffering larger nuisance costs reduce low types' virtual nuisance costs, diminishing the well-known downward distortion in low types' content allocation. Therefore, type-dependent ad nuisance can generate tension between welfare and consumer surplus.

Related Literature Our paper enriches the literature on business models of media platforms. Here we discuss several recent papers that are close to ours.

Sato (2019) considers a model that is more general than ours on the advertising side but simpler on the content side. On the advertising side, there is a continuum of heterogeneous advertisers. But on the content side, the platform offers the same content to all consumers as they have the same value for the content, although they are heterogeneous in their ad nuisance costs. In such a model, he shows that the optimal menu is indeed binary that the platform offers only two services, and consumers are segmented into two groups: those who pay premium fees and those who view advertisements with no explicit fees. Our paper complements Sato (2019) as we focus on the design problem of allocating different sets of content to different consumer types, which he did not study.

Lin (2020) considers a monopoly platform that practices second-degree price discrimination vis-a-vis heterogeneous consumers of two types of quality taste. He offers a far more specified model for advertisers' matching with consumers and emphasizes the role of so-called *type-dependent externalities*: the subscription decision on the consumer side affects the matching probability between advertised products and targeted consumers, which in turn affects the consumer's subscription decision. In our model, we dramatically simplify this cross-side externality; instead, we focus on how advertising and the non-negative price constraint interplay with the standard trade-off between rent extraction and efficiency.[7]

Carroni and Paolini (2020) also consider a parsimonious model in which a monopoly platform intermediates users with advertisers and content providers. Users value content

[7]Our paper complements to Lin (2020). For example, in our simple model, bottom-up allocations for the low-type consumers can arise in the second-best optimum, which is not characterized in Lin (2020). He studies a different research question of how price discrimination on the consumer side complements or substitutes price discrimination on the advertiser side.

variety (for example, the number of songs available in Spotify) and service quality, but they dislike advertising ads. Their model predicts that a platform with a sufficiently large audience will offer a premium service only. Our model focuses on which set of content that would be offered to different types of consumers via screening contracts and self-selection.

Roadmap We progressively construct the paper. After introducing the baseline model in Section 2, we analyze the benchmark case of the subscription-based contract without advertising in Section 3 and the ad-funded contract in Section 4. In both sections, we study the first-best and the second-best contract. Section 5 introduces the non-negative price constraint and studies the freemium contract. Section 6 discusses several extensions of the baseline model by relaxing some assumptions. Section 7 concludes the paper.

2 The Model

There is a unit mass of consumers and a monopoly platform providing with or linking to content, possibly with advertising. Content $x \in X$ has two-dimensional attributes: quality $q(x)$ and ads-revenue $r(x)$. Assume that the entire set of content is described as a square in the space of (q, r), that is, $(q, r) \sim G \in \Delta[0, 1]^2$ where G is the joint distribution function.[8] For example, the content indexed by $(1, 1)$ means the highest quality content to consumers and the highest ad revenue generating to the platform. On the opposite extreme, the content at $(0, 0)$ means the lowest quality to consumers and the least profitable ads to the platform. Figure 1 illustrates how content x is described as a point in the two-dimensional attribute square.

Consumers are heterogeneous in their tastes for quality, measured by $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$, with a cumulative distribution function $F(\theta)$ and a density function $f(\theta) > 0$ on $[\underline{\theta}, \bar{\theta}]$. If we consider only two types of consumers, then $\theta \in \{\theta_L, \theta_H\}$ with $\theta_H > \theta_L$ and $\lambda := f(\theta_H)$ and $1 - \lambda = f(\theta_L)$ where λ denote the probability that consumer i is of a high-type.

A consumer θ earns the gross utility θq from consuming content of quality q. We assume that she faces an attention cost $a \geq 0$ per unit of content she consumes. This attention cost reflects the opportunity cost associated with enjoying the content.

The platform serves its content with a fixed amount of advertisements or offers it ads-free. All content is already produced and there is no cost of serving it to a consumer. A

[8]Given set S, we write ΔS for the set of all probability distributions on S. So, $\Delta[0, 1]^2$ captures the set of all joint distributions over the unit square comprising of a unit interval of q and a unit interval of r.

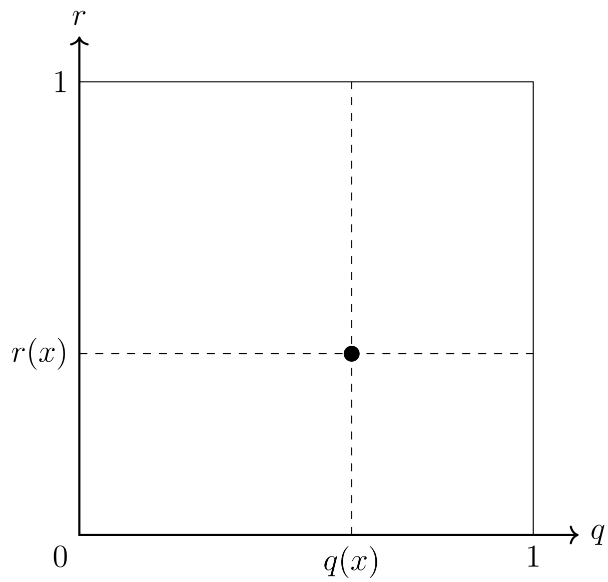


Figure 1: Content x has two attributes of quality q and ads-revenue r.

unit of content with advertising generates ad revenue of $r \in [0,1]$ for the platform. On the other hand, each ad imposes a nuisance cost of $c \geq 0$ proportional to content quality on consumers. The multiplicative specification builds upon the premise that a consumer is more bothered when interrupted by ads for more exciting content.For a meaningful analysis, we assume $c < \underline{\theta}$ to guarantee that the net marginal value of content remains positive for every consumer; otherwise, the low-type consumers would never consider any content regardless of the quality.

Let superscript k denote the ad-free content by $k = 0$ and the content with ads by $k = 1$. Let $S_\theta^k \subseteq X$ denote the set of content that a consumer of type θ consumes. We refer to the set of content as *content allocation*. A *contract* specifies a pair $\left(S_\theta^k, p_\theta^k\right)$, where $p_\theta^k \in \mathbb{R}$ is the price for consuming the allocation $S_\theta^k \subseteq X$. A negative price means a monetary compensation offered to a consumer for consumption. We assume that, before consumption, consumers do not observe the quality of individual pieces of content. Instead, they form a correct belief about the expected quality of all the content given the allocation she will consume.[9]

- A content allocation S is said to be *top-down* if there is $\hat{q} : [0,1] \to [0,1]$ such that $S = \{x \in X : q(x) \geq \hat{q}(r(x))\}$. For a fixed level of advertising revenue, a top-down

[9]Section 6 briefly discusses the opposite scenario in which consumers perfectly observe the quality of each individual piece of content.

allocation trims off the content whose quality does not exceed some threshold.

- A content allocation S is said to be *bottom-up* if there is $\hat{q} : [0,1] \to [0,1]$ such that $S = \{x \in X : q(x) \le \hat{q}(r(x))\}$.
- A content allocation S is said to be *shut-down* if $S = \emptyset$ and *full* if $S = X$, respectively.

Payoffs A consumer of type θ earns the gross utility $U_\theta(S^k_\theta)$ from consuming an allocation $S^k_\theta \subseteq X$:

$$\begin{aligned} U_\theta(S^k_\theta) &= \iint_{S^k_\theta} (\theta - \mathbb{1}_{\{k=1\}} c)q - a \, dG \\ &= (\theta - \mathbb{1}_{\{k=1\}} c)Q(S^k_\theta) - aN(S^k_\theta) \end{aligned} \tag{1}$$

where $\mathbb{1}_{\{k=1\}}(x)$ is the indicator function such that $\mathbb{1}_{\{k=1\}}(x) = 1$ if $x \in S^1_\theta$ and $\mathbb{1}_{\{k=1\}}(x) = 0$ if $x \notin S^1_\theta$. Since the platform can earn ad revenue, the welfare generated from a consumer of type θ_i is given by

$$W_\theta(S^k_\theta) = U_\theta(S^k_\theta) + \iint_{S^k_\theta} \mathbb{1}_{\{k=1\}} r \, dG = (\theta - \mathbb{1}_{\{k=1\}} c)Q(S^k_\theta) - aN(S^k_\theta) + R(S^k_\theta) \tag{2}$$

where, for simple notation, let $Q(S^k_\theta)$ denote the gross quality of all content $x \in S^k_\theta$ that a consumer of type θ obtain under advertising scheme k. Similarly, $N(S^k_\theta)$ denotes the content mass (number) in allocation S^k_θ. The platform's profit from advertising is $R(S^k_\theta)$.

Applications Our concepts of contracts and allocations reasonably capture many real-world applications. For example, consider the following cases:

1. Ads-free Premium Plan, $(S^1_L = S^0_H = X,\ 0 = p^1_L < p^0_H)$: There is no exclusive content for the high-type, but there is no ads for the high-type who pays a subscription fee for a premium service. Many media platforms such as *YouTube, Spotify, Hulu*, etc. adopt this kind of contract.

2. All-access Premium Plan, $(S^1_L \subset S^1_H = X,\ 0 = p^1_L < p^1_H)$. Both subscribers and non-subscribers are exposed to ads.

    - Top-down allocations: The non-subscribers face a restriction of access so that consumers end up choosing the high-quality content only for their consumption.

This contract appears to be consistent with the business models of the *New York Times* and the *Washington Post*.

- Bottom-up allocations: The non-subscribers cannot access the exclusive content, only available to the high-type subscribing to the service for a fee. This contract is found in *Le Monde* and the *Figaro*.

3 Benchmarks: Subscription-based contracts

We first briefly analyze subscription-based contracts without advertising. The contracts are specified as (S^0_θ, p^0_θ). The first-best arises under full information about θ, which serves as a benchmark to the second-best that arises when the content provider cannot observe consumer types. For the second-best, the content provider has to induce self-selection, following the revelation principle. As the standard analysis unfolds, we confirm that there will be no distortion for the high-type consumers but a downward distortion for the low-type to minimize the information rent yielded to the high-type.[10]

3.1 First-best subscription contracts

If the intermediary can observe the type θ of the consumer, it will solve

$$\max_{(S_\theta,\, p_\theta)} p_\theta \quad \text{subject to} \quad IR_\theta :\ U_\theta(S_\theta) - p_\theta \geq 0, \quad \forall \theta \in \Theta.$$

The platform will set the price $p_\theta = U_\theta(S_\theta)$ to extract all consumer surplus. The above problem becomes

$$\max_{S_\theta} U_\theta(S_\theta) = \theta Q(S_\theta) - aN(S_\theta).$$

The platform finds it profitable to serve any content x as long as its benefit $\theta q(x)$ exceeds the attention cost a that it imposes on the consumer, i.e., $\theta q(x) - a > 0$. This implies that all types of consumers face top-down allocations. Since $0 < a/\bar{\theta} < a/\underline{\theta}$, the set of content rendered to a lower type consumer is more restricted than the set of content offered to a higher type. Intuitively, the high-type consumers have a higher willingness to pay for a given allocation and therefore the platform includes more content in the package targeted to the high-type consumers with a higher price tag. All consumers have zero surplus as they

[10]For brief notations, we do not specify the superscript 0 in this section unless it is required for clear distinction.

pay up to their willingness to pay for a given content allocation.

3.2 Second-best subscription contracts

When the platform cannot observe the consumer types, the platform maximizes its profit, which is $p(\theta)$ if the consumer proves to be type θ. This profit needs to be multiplied by the number (probability) of type-θ consumers, which can be measured by $f(\theta)$. Hence, the (expected) profit is

$$\Pi = \int_{\underline{\theta}}^{\bar{\theta}} p(\theta) f(\theta) d\theta$$

Because the scheme must be feasible, it must satisfy both (IR) and (IC) constraints. Consider two arbitrary types, θ and θ' and take $\theta' > \theta$. Incentive compatibility requires that θ' does not want to purchase the package intended for θ and *vice versa*.

$$\begin{aligned} U_{\theta'}(S_{\theta'}) - p(\theta') &\geq U_{\theta'}(S_\theta) - p(\theta) \\ U_\theta(S_\theta) - p(\theta) &\geq U_\theta(S_{\theta'}) - p(\theta') \end{aligned}$$

Adding the two revealed-preference arguments, we have the single-crossing property:

$$U_{\theta'}(S_{\theta'}) - U_{\theta'}(S_\theta) \geq U_\theta(S_{\theta'}) - U_\theta(S_\theta) \tag{3}$$

The gain from the higher quality is bigger for the high-type consumers than that for the low-type consumers. Inequality (3) leads to the monotonicity constraint $Q(S_{\theta'}) \geq Q(S_\theta)$. In other words, a necessary condition for second-degree subscription pricing to be feasible is that $Q(\cdot)$ be non-decreasing. With the typical algebra and calculus on the second-degree price discrimination with a continuum of consumers, we can derive the second-best subscription profit as

$$\Pi = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \theta - \frac{1 - F(\theta)}{f(\theta)} Q(S_\theta) - aN(S_\theta) \right\} f(\theta) d\theta \tag{4}$$

Let $\theta^v := \theta - \frac{1-F(\theta)}{f(\theta)}$ be the virtual value of consumer θ. With the increasing hazard rate assumption, θ^v is increasing with θ. Regarding the content served to θ, it is still optimal to serve any content x such that $\theta q(x) - a \geq 0$.

Comparing the profit expressions, there is no allocation distortion for the highest type $\bar{\theta}$. On the other hand, for all other types $\theta < \bar{\theta}$, the virtual valuation is smaller than its original valuation θ. The platform serves the low-type consumers with content x if $\theta^v\, q(x) - a \geq 0$.

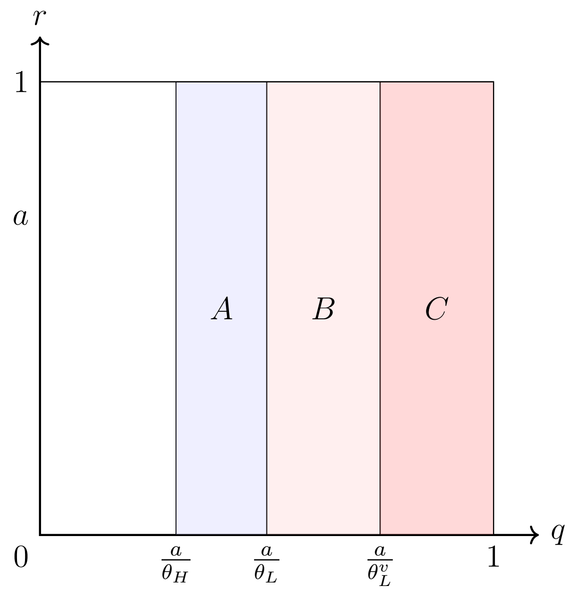


Figure 2: Subscription contracts with no advertising: first-best and second-best allocations

There are two distinct cases depending on the sign of the virtual valuation. Consider the case of $\theta^v < 0$. Since it leads to $\theta^v q(x) - a < 0$ for any non-negative a, the platform will not serve such a type: shutdown allocation is optimal. By contrast, if θ^v is positive, the content with quality index $q(x)$ being greater than a/θ^v will be provided.

Intuitively, when the platform does not involve advertising in a contract, the standard results — no distortion at the top and a downward distortion at the bottom — prevail in our vertically differentiated content space. However, what is important to notice is that the platform continues to apply top-down allocations to both types. Hence, the downward distortion manifests in the exclusion of low-quality content.

Figure 2 describes the allocations under the first-best subscription contracts and confirms the above results illustratively for the two types. The allocation with $A+B+C$ is offered to the high-type consumers under both first- and second-best contracts. On the other hand, the allocation with $B+C$ is offered to the low-type consumers under the first-best contract, and the allocation with only C is offered to the low-type consumers under the second-best. Hence, the allocation of B is excluded for the purpose of preventing the low-type consumers from mimicking the high-type consumers.[11]

Remark 1. Informational bundling plays no role in subscription contracts. Even if the qual-

[11]Note that in the real-world applications, the higher quality may not only mean the greater set of content allocations, but also more features that may reduce consumers' attention cost.

ity of each piece of content is perfectly observable, consumers will consume all the content they receive from the platform regardless of their types, both in the first-best and the second-best. For this reason, subscription prices are always positive.

4 Contracts with advertising

Now we study our primary topic of interest, ad-funded contracts in which consumers are exposed to advertising when consuming digital content. As in the previous section, we study the first-best and proceed to the second-best, where asymmetric information calls for an optimally designed menu. Contrary to what happens in the subscription contracts, we will see that information bundling plays a role, which may induce the platform to subsidize consumption through negative prices. Furthermore, the second-best may involve a bottom-up allocation, which was never the case in the subscription contracts. We here assume that the platform can subsidize consumption with a negative price, which will be relaxed when we study the optimal freemium contracts in Section 5.

4.1 First-best with advertising

If the platform can observe the type θ_i of the consumer, it will solve

$$\max_{(S_\theta, p_\theta)} \; p_\theta + R(S_\theta) \quad \text{subject to} \quad U_\theta(S_\theta) - p_\theta \geq 0, \quad \text{for } \forall \theta \in \Theta.$$

where $R(S_\theta) = \iint_{S_\theta} r dG$ and $R'(S) = r$. As before, the first-best allocations will be set to maximize the total welfare, and the price will be chosen to extract all consumer surpluses. Thus, $p_\theta = U_\theta(S_\theta)$, and the welfare maximization problem turns to

$$\max_{S_\theta} \,(\theta - c)Q(S_\theta) - aN(S_\theta) + R(S_\theta)$$

Intuitively, if a certain content does not generate a large enough revenue r compared to the attention cost a, then the welfare maximization requires that the quality must be high enough to compensate for this loss. Specifically, if $r < a$, only content with quality $q(x)$ higher than $(a - r)/(\theta - c)$ is served. Otherwise ($r > a$), all content regardless of the quality will be served. Put differently, for any content with $r < a$, there will be a top-down allocation. Figure 3 illustrates the first-best contracts with ads. As θ decreases, the line out of $(0, a)$ begins to move counter-clockwise and thus the set of the first-best allocation shrinks. More content is trimmed off in the top-down allocation.

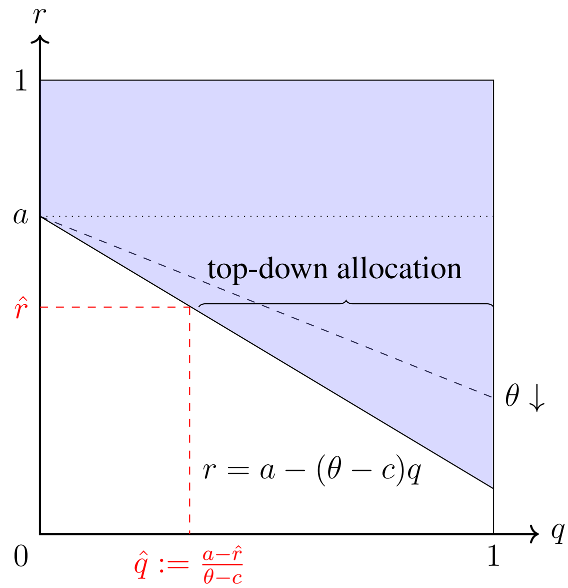


Figure 3: First-best contracts with advertising

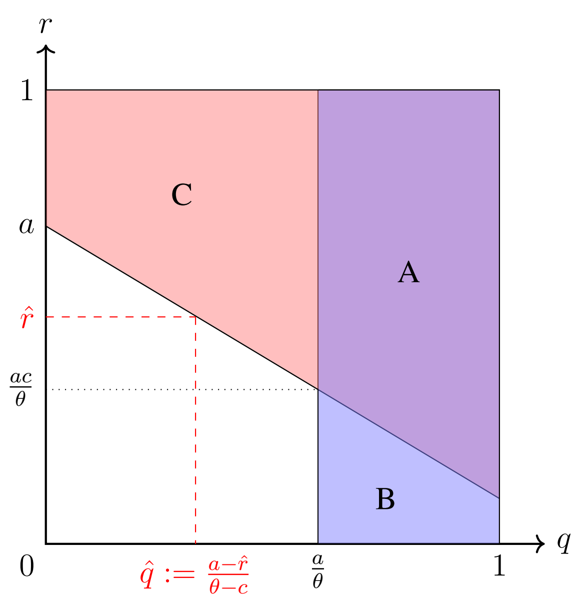


Figure 4: Comparing first-best contracts

Let us compare the first-best advertising contracts with the benchmark subscription contracts. First, as Figure 4 illustrates clearly, the allocations under the two regimes are different: $(A+B)$ under the subscription contract and $(A+C)$ under the advertising contract. Thus, the (relatively) low quality, high ads-revenue content, allocation C, is offered under the advertising contract. Since the profitability of a subscription contract does not depend on the ads-revenue r, higher quality content ($q(x) > a/\theta$) must be included in the package. As a result, the allocation of B is included in the subscription contract but not in the advertising contract.

Relative to the subscription contract, advertising expands the set of content by C but excludes the content by B.[12] For a fixed per unit attention cost, a consumer will weigh in the average quality of the content. As the platform keeps adding lower-quality content, the average quality may decrease, so further increasing consumption requires subsidizing consumption with a negative price, which we allow in this section.[13]

Remark 2. To see the role of informational bundling with advertising, suppose that consumers can avoid the consumption of any content that does not provide enough utility to compensate for the attention cost it imposes. Then, type-θ consumer will consume an advertised content x only if $(\theta - c)\, q(x) - a > 0$. Assume that $\theta - c > 0$.[14] Then, consumption of x requires its quality to be larger than $a/(\theta - c)$. A restriction over the quality of x is equivalent to a restriction over x itself, that is, a restriction over the choice set. Consequently, if content cannot be bundled, advertising cannot expand the set of content consumed in the first-best.

4.2 Second-best with advertising

As it occurs in the second-best of the subscription contract, the platform's second-best advertising contract is analogous to its first-best one with two differences. First, θ's valuation is replaced by its virtual counterpart $\theta^v := \theta - \frac{1-F(\theta)}{f(\theta)}$. Second, self-selection requires the

[12] If $c = 0$, then advertising will expand the content without excluding any content.

[13] This assumption is relaxed in the next section, where we analyze freemium contracts.

[14] It is immediate that if $\theta - c < 0$, no content is consumed by consumers of type θ.

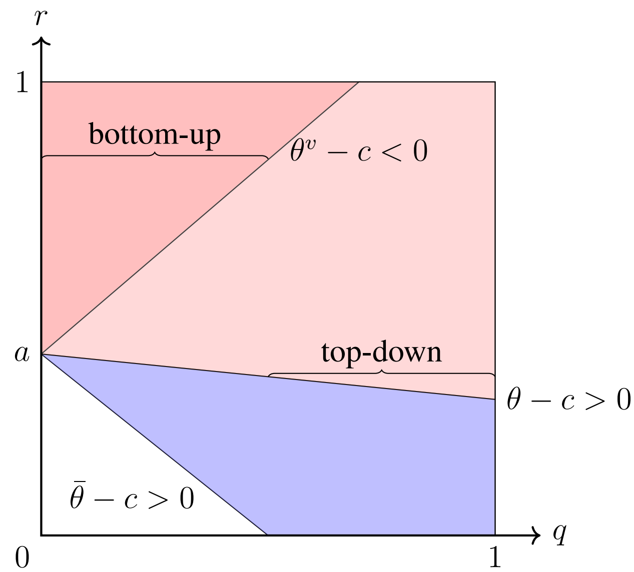


Figure 5: The second-best contracts with advertising

monotonicity condition. Hence, the platform solves[15]

$$\Pi = \int_{\underline{\theta}}^{\bar{\theta}} \{\theta^v - c)Q(S_\theta) - aN(S_\theta) + R(S_\theta)\} f(\theta)d\theta \tag{6}$$

Again, there is no allocative distortion for the high-type consumers. However, the low-type consumers' allocation may entail a bottom-up allocation by trimming off the high-quality content from the package.

The intuition is in what follows. The term $(r - a)$ captures a net profit from one unit of content served. Thus, if $r - a > (<)0$, the platform can increase its profit by adding (dropping) some content to the package. On the other hand, the term $\theta^v - c$ captures a net profit per quality. If $\theta^v - c > 0$, the profit increases with the quality of the content served. However, for $\theta^v - c < 0$ the platform would benefit by lowering the quality. Hence, depending on the signs of the two terms, we have four different situations as we summarized in Table 1.

The interesting and novel kind of allocation with advertising arises when ad revenue is

[15]If we consider the two types of consumers, then the above expression will be given as follows:

$$\max_{S_L, S_H} (1-\lambda)\left[(\theta_L^v - c)Q(S_L) - aN(S_L) + R(S_L)\right] + \lambda\left[(\theta_H - c)Q(S_H) - aN(S_H) + R(S_H)\right] \tag{5}$$

subject to (3).

Allocations	$r-a>0$	$r-a<0$
$\theta^v - c > 0$	full	top-down
$\theta^v - c < 0$	bottom-up	shutdown

Table 1: Various allocations in second-best contracts with advertising

high ($r-a>0$) and the virtual valuation of type θ is smaller than nuisance cost ($\theta^v - c < 0$). Note that as we assume $\theta - c > 0$, low-type consumers obtain positive utility from consumption gross of attention cost no matter the quality of content. However, $\theta^v - c < 0$ means that this utility is smaller than the information rent given to high-type consumers. Therefore, the platform serves a set of content to low-types to obtain ad revenue, but this is done to minimize information rent, making bottom-up allocations optimal. Specifically, only content of quality lower than $(a-r)/(\theta^v - c)$ is served.

Proposition 1 (Contracts with advertising). *Suppose that the platform can subsidize content consumption with a negative price. For advertising contracts, the following results hold:*

(a) In the first-best contract, the platform offers a full allocation for $r \geq a$ and a top-down contract for $r < a$ such that $\frac{a-r}{\theta-c} \leq q(x) \leq 1$. The smaller size of content allocation is offered to the lower-type consumers.

(b) In the second-best contract, the highest type consumer $\bar{\theta}$ receives her first-best allocation and earns a positive utility due to the information rent. The allocation never involves a bottom-up allocation.

(c) However, consumers of type θ receive a bottom-up allocation if $\theta^v < c$.

Proposition 1-(c) hints at a source of various real-world contracts. Some platforms offer bottom-up allocations. They limit the lower-type consumers' access to exclusive (apparently, high-quality) content.

Let us explain this result from the perspective of a contribution to the theoretical literature on monopolistic screening. Though we closely follow the Mussa-Rosen type monopolistic screening model, our model differs from it because we allow for a joint decision of quantity and quality by choice of subset through content allocation. This new consideration leads to richer results in the second-best.

In our model, downward distortion can take two different forms. First, it may entail a restricted quantity with smaller top-down allocations. Second, more interestingly, it may entail a restricted quality with bottom-up allocations. The change from top to bottom allocations with advertising has never been reported in the literature. The novel finding comes from the combination of the two features. We view a media platform's menu from a set of vertically differentiated content, and contracts involve content allocations.

5 Freemium contracts

There can be a case where the platform may be willing to subsidize consumption such that the second-best advertising contract specifies a negative price. However, such a negative price is subject to either an adverse selection problem or a moral hazard problem. The adverse selection problem means that a negative price may induce consumers who have no genuine interest in the content to pretend to have an interest only to receive the subsidy. The moral hazard problem implies that, after receiving the subsidy, consumers consume only a subset of the content allocated.

To avoid these problems, the platform needs to ensure that consumers get a non-negative gross utility from consuming the content allocated. This additional constraint, which we call an *ex post moral hazard constraint*, leads to the so-called "freemium strategy". Some content with advertising is provided free of charge to some low-type consumers, while full content is provided to the high-type consumers at a subscription fee (with or without advertising). One theoretical contribution to the field is that we offer the first theoretical analysis of the freemium contracts as an optimal mechanism design over content allocation and pricing.

We denote content allocations in a freemium contract by S_θ^f. Suppose $p_\theta = U_\theta(S_\theta) < 0$ under the second-best contract with advertising. This means that the unconstrained second-best advertising contract demands a money transfer from the platform to the low-type consumers. Then, the optimal freemium allocation S_θ^f must satisfy $U_\theta(S_\theta^f) = 0$, that is,

$$\begin{aligned} 0 &= (\theta - c)\, Q(S_\theta^f) - a\, N(S_\theta^f) \\ &= \left[(\theta - c)\, \frac{Q(S_\theta^f)}{N(S_\theta^f)} - a \right] \times N(S_\theta^f). \end{aligned} \tag{7}$$

5.1 Optimal freemium contracts

For Equation (7) to be satisfied, it follows either $S_\theta^f = \emptyset$ or S_θ^f have average quality equal to $\frac{Q(S_\theta^f)}{N(S_\theta^f)} = \frac{a}{\theta - c}$. Suppose $S_\theta^f \neq \emptyset$. Then the platform must add to its second-best problem a new restriction that the average quality of S_θ^f is equal to $a/(\theta - c)$. That is, the platform solves

$$\max_{S_\theta^f} \int_{\underline{\theta}}^{\bar{\theta}} \{\theta^v - c)Q(S_\theta) - aN(S_\theta) + R(S_\theta)\} f(\theta) d\theta$$

$$\text{subject to} \quad \text{(i) ex post moral hazard constraint:} \quad \frac{Q(S_\theta^f)}{N(S_\theta^f)} = \frac{a}{\theta - c}$$

$$\text{(ii) monotonicity constraint:} \quad Q'(S_\theta^f) \geq 0.$$

Ignoring constraint (ii) and substituting constraint (i) into the first bracketed term of the objective function, we have

$$\begin{aligned}
(\theta^v - c)\, Q(S_\theta^f) - aN(S_\theta^f) + R(S_\theta^f) &= \left((\theta^v - c)\, \frac{Q(S_\theta^f)}{N(S_\theta^f)} + r^f - a \right) N(S_\theta^f) \\
&= \left((\theta^v - c) \frac{a}{\theta - c} + r^f - a \right) N(S_\theta^f) \\
&= \left(r_\theta^f - a\, \frac{\theta - \theta^v}{\theta - c} \right) N(S_\theta^f).
\end{aligned}$$

where r^f is the average revenue of the freemium allocation, that is, $r_\theta^f := R(S_\theta^f)/N(S_\theta^f)$.

Hence, the platform needs to find the freemium allocation that maximizes

$$\max_{S_\theta^f} \left(r_\theta^f - a\, \frac{\theta - \theta^v}{\theta - c} \right) N(S_\theta^f).$$

There are two cases to be considered. First, if $\frac{r_\theta^f}{a} < \frac{\theta - \theta^v}{\theta - c}$, then $S_\theta^f = \emptyset$. Second, if $\frac{r_\theta^f}{a} > \frac{\theta - \theta^v}{\theta - c}$, then the platform must choose an allocation that has maximum measure among the allocations with the required average quality. Recalling $\theta^v := \theta - \frac{1 - F(\theta)}{f(\theta)}$, the two cases depend on whether r^f is sufficiently large enough that

$$r_\theta^f \geq r^*(a, c, \theta) := \frac{a}{\theta - c} \frac{1 - F(\theta)}{f(\theta)}. \tag{8}$$

Since r_θ^f is increasing in each of (a, c), the shut-down allocation for type-θ consumer is more likely when the attention cost (a) is higher or the disutility from the ads (c) is higher. We summarize our findings as follows:

Proposition 2 (Freemium contract). *The optimal freemium contract assigns θ-type consumers an allocation S_θ^f such that* *(a) If* $r_\theta^f < \frac{a}{\theta-c}\frac{1-F(\theta)}{f(\theta)}$*, then shutdown allocation is offered.*

(b) If $r_\theta^f > \frac{a}{\theta-c}\frac{1-F(\theta)}{f(\theta)}$*, then the optimal offer is to provide the largest measure whose average quality is equal to* $a/(\theta-c)$.

5.2 When do freemium contracts make consumers better?

Because any freemium contract imposes an additional constraint on the platform's optimization problem, the profit from the freemium contract cannot be higher than the profit from the second-best ad-funded contract. How would such a freemium contract affect consumer welfare?

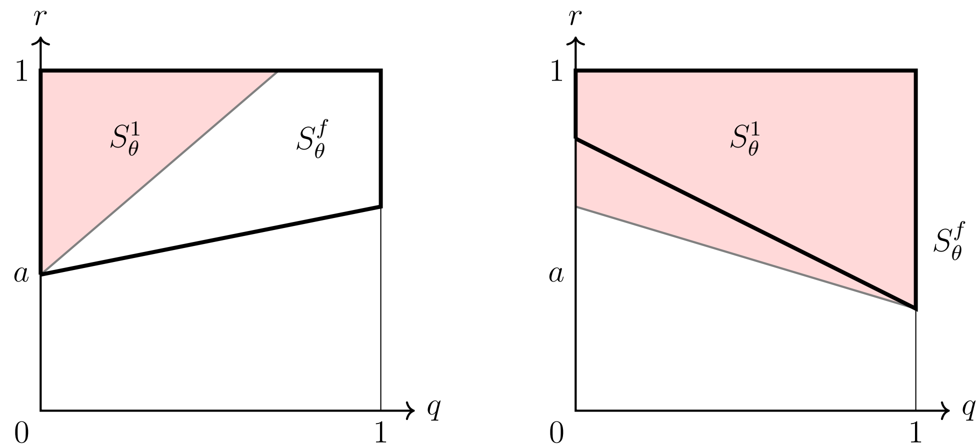
First of all, if $\frac{\theta-\theta^v}{\theta-c} > 1$, which is equal to $\theta^v - c < 0$, as we can see in Figure 6a, the unrestricted second-best allocation denoted by S_θ^1 is a bottom-up allocation. If initially, the overall quality is too low, the content provider will offer a larger bottom-up allocation, adding content (to maximize the measure of content consumed) until it reaches the minimum average quality required. That is, $S_\theta^f \supset S_\theta^1$ where the new allocation S_θ^f is the trapezoid with a thick line.

For intuition's sake, think of the two types of consumers. Note the important fact that the low-type consumers will receive zero surplus in any optimal contract and that the high-type consumers' surplus is proportionally increasing with the total quality measure for the low-type consumers as the information rent yielded to the high-type consumers is increasing with it. This implies that we can also say that when the ex post moral hazard constraint binds, it increases consumer surplus if the low-type consumers receive a bottom-up allocation and thus a larger total quality despite the equal average quality.

On the contrary, if $\theta_\theta^v > c$, as in Figure 6b, S_θ^f will be top-down allocations with $S_\theta^f \subset S_\theta^1$. Intuitively, if the overall quality is too low, the platform will remove some of the lowest-quality content of the initial bundle until the average quality reaches the minimum level required. Hence, when the ex post moral hazard constraint binds, this reduces consumer surplus if the low-type consumers receive top-down allocations and thereby a smaller total quality measure.

Proposition 3. *(a) If* $r_\theta^f > \frac{a}{\theta-c}\frac{1-F(\theta)}{f(\theta)}$ *and* $\theta^v - c < 0$*, then consumer welfare increases with any binding ex post moral hazard constraint.*

(b) If $r_\theta^f > \frac{a}{\theta-c}\frac{1-F(\theta)}{f(\theta)}$ *and* $\theta^v - c > 0$*, then consumer welfare decreases with any binding ex post moral hazard constraint.*



(a) Bottom-up extension

(b) Top-down shrinkage

Figure 6: Freemium contracts with binding ex post moral hazard constraint

The information bundling plays an important role when the freemium contract is of a top-down allocation. This is because adding any extra below-average quality content to the initial set induces consumers to opt for zero consumption when the new content sells as a separate item, while the entire new set of content can be sold as a part of optimal freemium contracts combined with other existing content.

By contrast, the information bundling plays no role when it applied to a bottom-up allocation. This is because adding extra above-average quality content to the initial set still can be sold even if each piece of new content sells separately.

6 Extensions

Here we discuss how our results are enriched by relaxing some assumptions imposed on the baseline model. This discussion is based on the two-type model for a better illustration (but all insights can extend to a continuum of types).

6.1 Type-dependent nuisance costs

In the baseline model, we have assumed equal ad nuisance costs ($c_L = c_H = c$). However, the ad nuisance costs may be higher for the high-type consumers, i.e., $\Delta c = c_H - c_L > 0$. Then, the information rent under subscription contracts and under ad-funded contracts are respectively given by $\Delta\theta\, Q(S_L^0)$ and $(\Delta\theta - \Delta c)\, Q(S_L^1)$. Low-types' virtual valuation under subscription contracts and under ad-funded contracts are respectively given by θ_L^v and

$\theta_L^v - c + \frac{\lambda}{1-\lambda}\Delta c$. This implies that the ad-funded contracts become more attractive to the platform, other things being equal, and that the platform increases the total quality allocated to low-types relative to the case of $\Delta c = 0$ in an ad-funded contract. Even if the increase in the total quality allocated to low-types is socially desirable, the type-dependent nuisance cost can induce the platform to excessively use ad-funded contracts mainly for rent extraction even when ad revenue is small.

To fix the idea, assume $r = 0$; so the platform will never use advertising in a first-best world. Consider now the second-best subscription-based contract and introduce the advertising only to the low-types without changing the content allocation. Then, the profit will change by $(-(1-\lambda)c_L + \lambda\Delta c)\ Q(S_L^0)$ where the fist term represents the nuisance cost of low-types and the second represents the reduction in high-types' information rent. If $\frac{\lambda}{1-\lambda}\Delta c > c_L$, then the platform has an incentive to introduce advertising, which is socially inefficient. Furthermore, this lowers the consumer surplus.

6.2 Heterogeneity in ad revenues

In the baseline model, we have assumed the same ad revenue ($r_H = r_L = r$) regardless of the consumer type. We may introduce heterogeneous ad revenue such that the high-type generate higher ad revenue for the platform than the low-type, i.e., $r_H > r_L$. However, following Jeon et al. (2021) we can envision that the ad revenue *net of privacy cost* can be lower for the high-type than for the low-type. In the second case, the platform is more likely to adopt no advertising to the high-type consumers and advertising is only applied to the low-type consumers. Such a consideration expands the constellation of primitive parameters for a freemium strategy to arise in optimum.

6.3 Convex attention cost

In the baseline model we assume a constant attention cost $c > 0$ per content. Let us here relax this assumption and consider convex attention cost: the cost of consuming a mass n of content imposes an attention cost of $C(n)$, with $C' > 0$ and $C'' > 0$. Given a consumer of type θ_i, the problem of finding the first-best ad-funded contract $\widehat{S}_i^1$ that maximizes welfare is written as

$$\max_{S_i^1 \in X} (\theta_i - c)\, Q(S_i^1) + r\, n(S_i^1) - C(n(S_i^1)). \tag{9}$$

For every $n \in [0,1]$, define

$$X_n = \{\text{measurable subsets of } X \text{ with measure equal to } n\}.$$

Hence, the maximization problem (9) can be rewritten in nested form as

$$\max_{n\in[0,1]} \left[\max_{S_n \in X_n} (\theta_i - c)\, Q(S_n) + r\, n - C(n) \right].$$

As n is kept fixed in the inner maximization problem, it is clear that if $\theta_i - c$ is positive (resp. negative), a solution S_n^* must maximize (resp. minimize) $Q(S_n)$.

Suppose the content provider starts by selecting the top-highest quality content available, and continue by picking the top-highest quality content not previously selected until it reaches a selection of mass n. The final selected allocation will be then $[1-n, 1]$ (top-down allocations), and clearly no other allocation of measure n can have larger quality. Therefore, $\max_{S_n \in X_n} Q(S_n) = Q([1-n,1])$. Analogously, minimizing $Q(S_n)$ leads to the selection of $[0,n]$ (bottom-up allocations), i.e., $\min_{S_n \in X_n} Q(S_n) = Q([0,n])$.

Hence, if $\theta_i - c > 0$, maximizing (9) is equal to

$$\max_{n\in[0,1]} (\theta_i - c) \int_{1-n}^{1} q(x)\, dx + r\, n - C(n).$$

If the problem admits an interior solution, it must satisfy $(\theta_i - c)\, q(1-n) = C'(n) - r$. Note that, from $(\theta_i - c) > 0$, q strictly increasing, and $C(\,\cdot\,)$ strictly convex, it follows that the LHS strictly decreases in n while the RHS is strictly increasing in n.

On the other hand, if $\theta_i - c < 0$, maximizing (9) is equal to

$$\max_{n\in[0,1]} (\theta_i - c) \int_{0}^{n} q(x)\, dx + r\, n - C(n)$$

and an interior solution must satisfy $(\theta_i - c)\, q(n) = C'(n) - r$. The LHS is strictly decreasing in n; the RHS is strictly increasing in n. Corner solutions correspond to either shutdown allocation or full allocation.

This extension confirms that all of our qualitative results under the constant attention cost can be preserved with a convex attention cost with some modification.

7 Conclusion

We analyzed a simple model of monopolistic screening in the context of a media platform and heterogeneous consumers who may pay a subscription fee for content or watch ads instead. A unique feature of our theoretical modeling is to consider a design of the menu in terms of content allocations: different packages may have different quality and quantity of content with different price tags or advertising volumes. Such a novel view provides a new insight even under the standard second-degree price discrimination as a mechanism design.

We find that advertising can change top-down allocations to bottom-up allocations. We also find that advertising can induce the platform to use informational bundling to expand the set of content consumed by consumers. When the non-negative price constraint binds under advertising, a freemium contract becomes optimal. The binding constraint reduces the set of content offered under top-down allocations relative to the absence of the constraint. Type-dependent nuisance costs under advertising generate an interesting trade-off between consumer surplus and welfare: when a high type consumer experiences more nuisance costs from advertising than the low types do, this improves welfare by expanding the set of content consumed by the low-type consumers, but it reduces the information rent of the high-type consumers and thereby the consumer surplus.

This paper provides a new perspective to understand the freemium strategy from a mechanism design approach with advertising. One may think of the freemium strategy as classic versioning under second-degree price discrimination. Some consumers choose to pay for the premium version with fewer or no ads, while others remain for the basic version with more ads to save the fee. Recently, this classic view was complemented with a two-sided market approach (e.g., Lin (2020) and Jeon et al. (2021)). The seemingly simple versioning strategy differs from other versioning. Online media platforms involve type-dependent cross-side externalities: consumers' menu choice affects the matching between eyeballs and advertised products, affecting advertisers' choices and, subsequently, users. In this paper, we do not rely on the cross-side externalities. Instead, the freemium strategy arises in our model when the platform involves both advertising and a non-negative price constraint. Thus, this paper further expands on how we can understand prevalent media platforms' freemium strategy.

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