Price-Match Guarantees and Investment Incentives

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Abstract

We consider duopoly sequential price competition between a low-cost online firm and a high-cost brick-and-mortar firm that decides whether to price-match the low-cost rival. We study how price-match guarantees affect the incentives of both firms to invest in cost reduction and quality enhancement. We find that price-match guarantees in our model weaken these incentives in most cases. Our research reveals more reasons to suspect that seemingly pro-competitive price-matching by many offline rivals to online sellers may have hidden social costs. (JEL codes: D4, L13, L4, M2)

Keywords: price-match guarantees, duopoly competition, investment incentives

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1 Introduction

Price-match guarantees (henceforth, PMGs) have been used by firms since at least 1947 (Edlin and Emch, 1999). Recent data show 12% of the top 500 retailers selling electronics and computers price-match, while 50% of the hardware and home improvement stores and 100% of office supplies stores commit to such policies (Jiang et al., 2016). To consumers, this may appear to be beneficial but economists, policymakers, and media have long debated over whether PMGs are pro-competitive or anti-competitive.¹

Recently, more attentions have been paid to so-called “showrooming” practices and how firms attempt to avoid potential free-riding by buyers at their expense. At large, showrooming can take place in two different contexts: 1) buyers wishing to analyze a product first-hand may visit a brick-and-mortar showroom, but then purchase the product online at a lower price; 2) buyers may search on an online marketplace ‘showroom’ to find the lowest-price or best-quality seller, then purchase the product directly from the seller or through a different platform at a lower price. Brick-and-mortar stores have used PMGs to address the first type of showrooming.² And, online platforms have used “price parity clauses” (PPCs) to deal with the second type.³ In both situations, one of the main issues is how PMGs or PPCs affect investment incentives of sellers and platforms.

In this paper, we use a standard Hotelling-type model of duopoly competition between a low-cost online firm and a high-cost brick-and-mortar firm and consumers with heterogeneous preferences for online versus offline shopping. In our model, the offline high-cost firm decides whether to adopt a PMG or not, after which both firms engage in the Stackelberg price competition with the online firm moving first. The model reflects reality in that usually higher-cost offline firms have

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¹See for example “Guaranteed profits: Price-match guarantees prevent rather than provoke price wars” The Economist 12 Feb. 2015.


³Wide PPCs prevent sellers from posting a lower price on any alternative sales channels. By contrast, narrow PPCs allow sellers to differentiate prices across platforms but prevents lower prices on the direct sales channel. See Edelman and Wright (2015); Wang and Wright (2016, forthcoming), and references therein for more detailed discussions on PPCs.
been price-matching their lower-cost online opponents.⁴

After we show how PMGs affect the online rival, the consumers, and social welfare (Proposition 1), we focus on its effects on investment incentives. We find a new type of inefficiency resulting from PMGs. First, we show that PMGs may dampen the high-cost offline firm’s investment incentive to decrease its cost disadvantage. Intuitively, there are two channels through which the high-cost firm’s incentive is affected: (i) a lower cost expands its market share through more competitive pricing, which we refer to as the extensive margin effect and (ii) it also raises the mark-up per unit of sales, which is the intensive margin effect. With PMGs, however, the high-cost firm already secures half of the market, and further market expansion from lower cost is not possible. Besides, the high-cost firm as the follower in a sequential price-setting passes the entire reduction in its cost to the consumers through a lower price, which means no intensive margin effect either. Consequently, on both accounts, the high-cost firm has weaker investment incentives for cost-reduction with PMGs (Proposition 2). We find a similar result for the low-cost firm, but the reason is slightly different. Again, PMGs make the extensive margin effect null. There is some positive intensive margin effect for the low-cost firm from its cost reduction, but the intensive margin effect is smaller with PMGs compared to that without PMGs. As a result, PMGs result in a low-cost firm to have a weaker investment incentive also (Proposition 3).

Second, we study firms’ incentives to invest in quality improvement. We find that PMGs weaken the high-cost firm’s investment incentive for a sufficiently small cost disadvantage. Why does the size of cost difference matter? The logic is as follows. Let \( c \) denote the cost difference.⁵ Under PMGs, the investment incentive is independent of \( c \) because the market share and the mark-up for the high-cost firm are independent of \( c \). By contrast, without PMGs, the intensive margin

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⁴For example, Best Buy adopted the following price-match policy, “At the time of sale, we [Best Buy] price-match all local retail competitors (including their online prices) and we price-match products shipped from and sold by these major online retailers: Amazon.com, Bphotovideo.com, Crutchfield.com, Dell.com, HP.com, Newegg.com, and TigerDirect.com.” https://www.bestbuy.com/site/help-topics/best-buy-price-match-guarantee/pcmcat297300050000.c?id=pcmcat297300050000 (Accessed on June 7, 2020)

⁵For reasons of simplification that we elaborate upon in Section 3, we normalize the low-cost firm’s cost to 0. Note that this implies \( \Delta c = c \), i.e., the cost of the high-cost firm is also the cost difference between the two firms.
effect and extensive margin effect both increase with $c$ if the low-cost firm’s quality improves. In other words, the investment incentives for quality are stronger when its cost disadvantage is bigger, which is intuitive. As a result, the high-cost firm’s investment incentive is smaller under PMGs for a small cost disadvantage, whereas the opposite is the case for a large $c$ (Proposition 4). Regarding the low-cost firm’s incentive to improve its quality, we find that PMGs unconditionally weakens the low-cost firm’s quality innovation incentives (Proposition 5). Under PMGs, the market share is independent of $c$; however, the intensive margin effect becomes negative because the price-match leads to an even lower mark-up when the quality difference enlarges. Without PMGs, both the intensive margin and extensive margin effects turn positive. As a result, PMGs do not increase investment incentives for the low-cost firm.

Our research reveals reasons to suspect that seemingly pro-competitive price-matching by many offline rivals to online sellers may have additional hidden social costs. These hidden social costs strengthen the case for policymakers to intervene in specific contexts and limit such practices when empirical evidence supports our theoretical findings.

2 Related literature

The early literature on price-matching emphasized the anti-competitive aspect of this practice. One plausible reason why a firm adopts a PMG is that it sends a message to its rivals that they will not be able to increase sales by cutting price because the undercut price will be matched and thus buyers cannot be poached away. As long as a firm can make such a promise credible, for instance, by putting up a written notice in its store or website, it ties itself to matching lower prices even in cases where it is harmful to the firm. Given the credibility of the threat, firms move towards an outcome of tacit collusion where they charge a price higher than they could have in equilibrium without price-matching. This idea was first put forward by Hay (1982) and Salop (1986) although
Cooper (1986) had earlier pointed out a similar effect of most-favored-customer clauses. Our paper is similar to this early strand in that firms price-match in equilibrium by having equal posted prices, obviating the need for consumers to invoke the price-match clause in equilibrium.

However, the anti-competitiveness of price-matching policies can be eliminated if firms adopt price-match guarantees and price-beating guarantees. There are no a priori reasons to preclude this given the lack of legal barriers against either practice. This may allow a firm to post a price higher than those of rivals and then offer to beat the rivals’ lower price by a certain amount. As long as the rivals are price-matching posted prices, consumers cannot invoke the price-match with rivals since the posted price of the firm with the price-beating policy is higher. Meanwhile, the price-beating firm lures customers away from the rivals by cutting actual prices below its rivals. This will lead to the rivals adopting the same price-beating policy, which in turn leads to non-collusive Bertrand price competition (Corts, 1997; Hviid and Shaffer, 1994). However, tacit collusion is restored when firms match both advertised and effective selling prices (Edlin, 1997; Kaplan, 2000; Arbatskaya et al., 2004), and Arbatskaya et al. (2006) provide data to show that in some cases firms do price-match actual prices on top of advertised prices. Given the preceding discussion and the paucity of cases where offline firms use price-beating guarantees, we restrict our attention to price-match guarantees only.

While tacit collusion has been the dominant theme to explain price-matching, more recent literature has emphasized a different reason why firms price-match. Moorthy and Winter (2006) and Moorthy and Zhang (2006) argue that in a duopoly with asymmetric costs, the low-cost firm uses a price-match guarantee to “signal” to uninformed consumers its low-price advantage over the high-cost rival. They show why the high-cost rival will not find it in its interest to adopt a PMG.

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6The working paper version of Cooper's paper is from 1981.

7Most-favored-customer clauses promise customers a refund of the price difference in case a firm lowers price after the buyer purchases the item. In essence, the firm matches its own future price rather than that of rivals. It has very similar potential implications for facilitating collusion. More recently, a variation of MFC clauses have appeared in the context of platforms – price parity clauses (PPCs).

8With price-matching, a firm will issue a refund to the customer that brings its price down to the price of a rival. With price-beating, the refund brings the firm’s price below that of the rival by a pre-specified amount, say 10%, because of an additional compensation more than the price difference.
This lack of incentive ensures only the low-cost firm price-matches, and the signaling mechanism works in equilibrium to aid buyers in identifying the low-cost from the high-cost firm. The issue with this narrative is that high-cost brick-and-mortar firms are increasingly adopting price-match policies that specifically match prices of low-cost online firms. Because our model does not depend on consumers to be informed about lower prices to benefit from the PMG, the need for signaling is moot.

Our paper is related to Edlin and Emch (1999) in that they show welfare losses arising not just due to higher prices as a result of price-matching, but these higher prices induce excessive entry by firms, which generate further welfare losses. Instead of looking at firm entry and exit, we attempt to uncover the second-order effects of price-matching practices on investment incentives. In terms of theoretical framework, our model is similar to that of Logan and Lutter (1989), who considered differentiated product duopoly markets with asymmetric costs. They find that the ability of price-match policies to sustain collusive prices depends on whether the high-cost firm chooses to price-match. If the firms’ costs are very different, the high-cost firm does not price-match, and pricing is competitive. On the other hand, if the firms’ costs are similar, the high-cost firm price-matches and pricing is above the competitive level. While we use a similar model to Logan and Lutter (1989), we address very different questions to uncover new effects of price-matching.

There is an extensive empirical literature on price-matching. Pertinent to our setup is a paper by Zhuo (2017). Using prices on online retailer Amazon before and after announcements of PMGs by traditional brick-and-mortar stores such as Walmart and Target, she finds that Amazon prices rose after such announcements. An interesting follow-up empirical exercise would be to find out whether such price-matching also led to both Amazon and Walmart/Target subsequently investing less in cost-reduction and quality upgrades.

PPCs and PMGs are similar in the sense that both result in price parity. However, PPCs are imposed by one firm (the platform) on another (the sellers on the platform), whereas PMGs are

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self-imposed by a firm on itself and results in no obligations upon anyone else, albeit they may induce others to react. Interestingly, Wang and Wright (2016) study the effects of showrooiming on a platform’s incentive to invest in advertising, increasing convenience benefits and reducing search costs for buyers and how PPCs alter those effects. They show that a monopoly platform will over-invest with narrow PPCs relative to the socially optimal investment. In contrast, we show that both online and brick-and-mortar firms may have weaker investment incentives with PMGs, compared to those without PMGs.

3 Model

We consider the standard Hotelling model of a duopoly. A continuum of consumers, whose mass is normalized to one, are uniformly distributed over the linear city $[0, 1]$. Each consumer has a unit demand. A brick-and-mortar seller, firm B, at the left end $(x = 0)$ and an online seller, firm O, at the right end $(x = 1)$ compete in prices. A consumer closer to firm B has a stronger preference for offline shopping relative to online shopping. Heterogeneous preferences regarding online vs. offline shopping may be attributed to different patience levels in waiting for an item to arrive from the online seller, the different utility of shopping online such as saving of time, less compulsive spending, no crowds relative to the inconvenience of the inability to physically examine before purchasing items, concerns about online scams and online privacy or security. The typical transport cost parameter $t$ measures the degree of the consumers’ relative preference intensity regarding the two different modes of shopping.\(^\text{10}\)

\(^\text{10}\)Consider two buyers with very different preferences. John likes crowds, but Jane does not. John is an outdoor person and likes driving to the store when he needs to; Jane likes to buy from the comfort of her home. John is impatient and hates waiting for shipments to arrive, whereas Jane is a good planner and anticipates her needs, so she usually orders early and hence does not get negatively affected by shipping times. While John likes to find out more about products by talking to people, Jane prefers reading about other people’s experiences with a product. In summary, John is at $x = 0$ and Jane is at $x = 1$. Other people with less extreme preferences are somewhere in the middle.

\(^\text{11}\)Lee (2006) consider a variant of the Hotelling model in which he models that a consumer pays online access cost of $a$ to capture search cost, uncertainty cost, order tracking cost, and delivery cost. Note that the online access cost affects the location of the marginal consumer, and thus the magnitude of intensive margin effects can depend on $a$. The online access cost would not change our main results qualitatively because we focus on the investment incentives which are not directly related to a fixed access cost.
Firm B’s cost of serving one consumer is \( c \in \mathbb{R}_+ \) whereas firm O’s is normalized to zero. We motivate the assumption of asymmetric costs by using the example of online vs. brick-and-mortar firms. It is generally true that online firms locate warehouses in remote areas with lower costs, which allows for carrying much greater inventory and for greater economies of scale. The need to bring in customers constrains brick-and-mortar retail stores to locate in high-traffic areas, and hence they face a cost disadvantage. Moreover, often online retailers operate marketplaces allowing them to offer goods through third-party sellers and which further reduces costs by eliminating the need to carry inventory.\(^\text{12}\) Online retailers also tend to more aggressively use technology to increase efficiency and cut costs\(^\text{13}\). A particular trend since online shopping became mainstream has been the closure of brick-and-mortar firms, big and small, as they failed to compete with online firms with a cost advantage such as Amazon.\(^\text{14}\)

Notice that online and offline firms may differ in their other features such as warranty, shipping, or return policies. For instance, brick-and-mortar stores may try to enhance in-store services by hiring more staff and offering pick-up services for online orders. So we introduce a quality difference to analyze the impact of price-matching on either firms’ incentive to invest in upgrading such quality dimensions like shipping speed. With \( v_O \) and \( v_B \) representing gross utility from consuming the good sold by firm O and B respectively, let \( \alpha = v_O - v_B \) so that \( \alpha \) denotes the online firm’s quality superiority if \( \alpha > 0 \) or quality inferiority for \( \alpha < 0 \), and \( \alpha = 0 \) representing the special case of the same quality.

We assume that the market is fully covered, and thus each firm is not a local monopoly. The timing of the game between the two firms is as follows. At the initial node, the high-cost firm B de-

\(^{12}\)While throughout the paper we use the online vs. offline example, our results naturally extend to cases where there is cost asymmetry between firms due to factors other than those that differentiate online and brick-and-mortar firms. The implications of cost asymmetry are important for our results, but the reason for the asymmetry is not.


cides whether to price-match or not.\footnote{In reality some online firms may price-match as well, e.g., eBay and NewEgg. We opted to study the case of the brick-and-mortar firm adopting a price-match, which has become more common lately.} We assume the decision to price-match or not is irrevocable. The sequence of deciding first to price-match or not and then choosing prices is reasonable since firms change prices frequently, whereas they tend to stick to price-matching (or not) for a longer period. The proposed timing also justifies our assumption of irrevocability as long as the duration of the game we study is shorter or equal to the period during which a firm does not change its price-matching policy. In the subgame in which firm B does not choose to price-match, we consider the sequential-move game\footnote{In Appendix B, we show that price-matching does not arise in a pure-strategy Nash equilibrium in the simultaneous-move game.} in which the low-cost firm O acts as the price leader and the high-cost firm B the follower. This sequential timing is consistent with Amir and Stepanova (2006) and Van Damme and Hurkens (2004). These papers show that in the presence of asymmetric costs, the low-cost firm finds it profitable to be the price leader, whereas firms prefer to be the price follower in the case of price competition between two symmetric firms. The equilibrium concept is the subgame-perfect Nash equilibrium.

### 3.1 No price-match subgame

Consider the subgame ensuing from firm B’s choice not to price-match. We characterize the demand for each firm by identifying the indifferent consumer’s location $x = \frac{1}{2} + \frac{p_O - p_B - \alpha}{2t}$, which gives the demand for B, $Q_B(p_B, p_O) = x$, and for O, $Q_O(p_B, p_O) = 1 - x$. The notation is self-explanatory: $p_B$ and $p_O$ are respectively B’s price and O’s price and $Q_B$ and $Q_O$ are demands. We start with B’s profit maximization problem: $\max_{p_B} \pi_B = (p_B - c)Q_B$ which renders the following best response function of $p_B(p_O) = \frac{1}{2}(t + p_O + c - \alpha)$. Firm B’s best-response plays the role of a constraint in firm O’s optimization problem:

$$\max_{p_O} p_O \left( \frac{1}{2} + \frac{p_B - p_O + \alpha}{2t} \right) \quad \text{s.t.} \quad p_B = \frac{1}{2}(t + p_O + c - \alpha).$$
Solving this problem, we derive the prices as follows:

\[ p^*_O = \frac{3t + c + \alpha}{2} \quad \text{and} \quad p^*_B = \frac{5t + 3c - \alpha}{4} \]  

which yields the location of the marginal consumer \( x^* = \frac{5t - c - \alpha}{8t} \).

In theory, there are two different cases depending on the primitive parameters \( c \) and \( t \). If \( c \leq t + 3\alpha \), then firm B’s price is lower than firm O’s, \( p^*_O \geq p^*_B \), where the equality holds for \( c = t + 3\alpha \). In this case, consumers find no need to claim the price-match—even if firm B committed to such a policy—as they find the online price is higher than the offline price. In contrast, if the offline firm’s cost disadvantage is large enough such that \( c > t + 3\alpha \), then \( p^*_O < p^*_B \). Let us assume \( c > t + 3\alpha \) for subsequent analyses to ensure that the firm O’s cost disadvantage is not so low that its price-match becomes irrelevant for subsequent price competition.

In the absence of firm B’s price-match, the pair of prices in (1) constitute the equilibrium prices. Each firm’s profit is computed as

\[ \pi^*_B = x^* (p^*_B - c) = (5t - c - \alpha)^2 \quad \text{and} \quad \pi^*_O = (1 - x^*) p^*_O = (3t + c + \alpha)^2 \]  

For firm B to gain a positive market share (i.e. \( x^* > 0 \)), we need \( c < 5t - \alpha \). Intuitively, this sets an upper bound to the cost disadvantage which, if breached, will imply firm B is not competitive enough to survive. Additionally, the full market coverage requires \( v_B - tx^* - p^*_B \geq 0 \), which is simplified as

\[ v_B \geq \frac{15t + 5c - 7\alpha}{8}. \]  

This outcome would occur at point E in Figure 1. (3) requires a sufficiently high gross utility from consuming B’s good for the indifferent consumer to be willing to purchase.
3.2 Price-match subgame

In the presence of the price-match commitment, the online seller’s price choice is bounded by a convex set composed of a 45-degree line from the origin to point M and then follows $BR_B$ beyond point M; the relevant edges are shaded in Figure 1. Firm O obtains its highest profit when its iso-profit curve passes through point M. Hence, the equilibrium prices are determined at the intersection of \( p_O = p_B \) and \( p_B = \frac{t + c}{2} \), which gives

\[
p_B^m = p_O^m = t + c - \alpha
\]  

(4)

where the subscript \( m \) indicates the price-match in consideration. At equal prices, each firm’s profit is computed as

\[
\pi_B^m = x^m (p_B^m - c) = \frac{(t - \alpha)^2}{2t}; \quad \pi_O^m = (1 - x^m) p_O^m = \frac{(t + \alpha)(t + c - \alpha)}{2t}
\]  

(5)
where \( x^m = \frac{t-\alpha}{2t} \). We assume \(|\alpha| < t\) for an interior solution. The sufficient condition for full market coverage is \( v_B - tx^m - p^m \geq 0 \), which is simplified as

\[
v_B \geq \frac{3t - 3\alpha + 2c}{2}.
\]

(6)

But note that we need to check when both firms do not deviate from the given price, \( p^m = t + c - \alpha \). Suppose that the online firm attempts to monopolize the market by deviating to \( p^d_O = \alpha + c - t \) so that the least interested consumer at \( x = 0 \) would prefer buying from firm O to buying from firm B, even if firm B follows and sets its most competitive price, \( c \). Then, firm O’s deviation payoff would be \( \pi^d_O = \alpha + c - t \). Thus, no deviation requires \( \pi^d_O \leq \pi^m_O \iff \frac{(t+\alpha)(3t+c-\alpha)}{2} \geq \frac{t+c}{2} \), which is the case if \( c \leq 3t + \alpha \).

In summary, we make the following parametric assumptions to focus on a relevant and interesting case of the game:

**Assumption 1.** We consider an interior solution where both firms have a positive market share instead of a corner solution where one firm covers the entire market, i.e., \(|\alpha| < t\) and \( c > t + 3\alpha \) and \( c \leq 3t + \alpha \). In addition, we consider a fully-covered market instead of local monopolies, i.e., \( v_B \geq \max \left\{ \frac{15t+5c-7\alpha}{8}, \frac{3t-3\alpha+2c}{2} \right\} \).

Note that for \( \alpha = 0 \), Assumption 1 collapses to \( v \geq \frac{3}{2}t + c \) and \( c \in (t, 3t] \). Because we obtain the same qualitative results when considering the simpler case of symmetric quality \( \alpha = 0 \), we only consider the general case with \( \alpha \neq 0 \) when we study the innovation incentives in Section 5.

### 3.3 Equilibrium and Static Effects

Under Assumption 1, the subgame perfect Nash equilibrium of the sequential pricing game is characterized by the path to the price-match subgame. The equilibrium prices and payoffs are given by (2) and (5) where \( \alpha = 0 \) is applied. Since firm B’s price-match decision depends on the relative size of the continuation payoffs \( \pi^*_B \) and \( \pi^m_B \), we can say that the price-match will be chosen
if
\[ \pi_B^m - \pi_B^* = \frac{t}{2} - \frac{(5t - c)^2}{32t} - \frac{(c - t)(9t - c)}{32t} \] \( > 0 \) \tag{7}
which is satisfied under Assumption 1. Graphically, it is clear from Figure 1 that firm B’s isoprofit curve passing through point M is above the one passing through point E.

We find how PMGs affect the online rival, consumers, and social welfare. In summary, we find:

**Proposition 1.** Price-match guarantees result in, compared to the no price-match benchmark, (i) the online firm being worse off, (ii) each consumer receiving a smaller net surplus, and (iii) a decrease in social welfare.

Since these do not constitute our main results, we provide all detailed analyses in Appendix A.

## 4 The incentive to invest in cost reduction

Here we extend the baseline model to study how PMGs would affect investment incentives. For this purpose, we let either firm make investments after firm B’s decision to offer a PMG but before price competition. We could explicitly model investment cost functions, but our analysis does not make it imperative. This is because essentially we end up comparing the marginal benefit of such investments with the price-match and without it, which means the marginal cost of investment is not crucial for analysis, unless if we were specifically interested in the optimal choice of investment level. Instead, we assume a one-time fixed cost \( F \) of investing in the cost-reduction technology and that the size of \( F \) does not depend on whether a PMG is offered or not.

### 4.1 Incentive for firm B

Consider first the investment incentive for firm B. Using the equilibrium payoffs we had earlier derived and after subtracting fixed cost \( F \), we have \( \pi_B^m = \frac{t}{2} - F \) and \( \pi_B^* = \frac{(5t - c)^2}{32t} - F \). With price-matching, the marginal change in firm B’s profit in response to a marginal decrease in its

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cost disadvantage is given by

\[
- \frac{d\pi^m_B}{dc} = x^m \left( 1 - \frac{\partial p^m_B}{\partial c} \right) + (p^m_B - c) \left( - \frac{\partial x^m}{\partial c} \right) = 0.
\]  

(8)

The first term in (8) measures the effect of a reduction in cost on the mark-up for each unit of existing sales. A decrease in the marginal cost by a dollar means an increase in profit due to the improved per-unit mark-up; however, its effect on the price is given by \(\frac{\partial p^m_B}{\partial c}\). Recalling that \(p^m_B = t + c\), the price goes up by the exact same amount. The two effects, the change in cost and the change in price, are exactly offset.\(^{17}\) The second term in (8) measures the extensive margin effect, which arises via a change in the market share. With the PMG, the extensive margin does not change because the firms must necessarily split the market in half at equal prices.\(^{18}\) In sum, we find both the intensive and extensive effects to be null, which leads to \(\frac{d\pi^m_B}{dc} = 0\). Firm B has no incentive to invest in cost-reduction under price-matching.

Next, we analyze firm B’s investment incentive following no price-match in a similar manner:

\[
- \frac{d\pi^*_B}{dc} = x^* \left( 1 - \frac{\partial p^*_B}{\partial c} \right) + (p^*_B - c) \left( - \frac{\partial x^*}{\partial c} \right) = \frac{5t - c}{16t} > 0
\]  

\(9\)

In contrast to the case of price-matching, there is a positive intensive margin effect without price-matching. The lower cost improves the mark-up since the market price does not increase as much as with a price-match, i.e. one-to-one. In addition, the cost-reduction has a positive effect on the extensive margin for firm B. With both intensive and extensive margin effects being positive, firm

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\(^{17}\)This is unique to the uniform distribution and in theory the intensive margin effect may go either way if other distributions are used to model consumer preferences.

\(^{18}\)The total market size is fixed with the Hotelling model. To consider a market expansion arising from the price levels, we may consider an alternative model such as a linear demand model with \(q_i(p_i, p_j) = a - bp_i + dp_j\) where \(a > c\) captures the market size, \(b > 0\) captures the demand sensitivity to the own price, and \(d\) captures the substitutability or complementarity between products. We can obtain qualitatively similar results of weakening investment incentives with this alternative setup. The Hotelling setup provides two advantages over this alternative specification: the computation and intuition associated with mathematical derivations are more clear, and it provides a micro-foundation for quality enhancement in Section 5.
B has a greater incentive to invest to reduce cost in a no-price-match regime.

**Proposition 2.** Under Assumption 1, price-matching by the high-cost firm eliminates its incentive to reduce its cost disadvantage.

For completeness of Proposition 2, we need to check more than local deviations. Recall that the condition to ensure \( p_B > p_O \) and hence price-matching takes place in equilibrium is \( c > t + 3\alpha \). A sufficiently large investment in cost reduction could bring \( c \) below \( t + 3\alpha \) and change the game from price-matching to no price-matching. However, since such a regime change lowers B’s profit, the argument in Proposition 2 can be extended to more than marginal changes in \( c \).

### 4.2 Incentive for firm O

We apply an analogous analysis for the online firm’s investment to expand its cost advantage by raising \( c \).\(^{19}\) Firm O’s equilibrium payoffs are \( \pi^m_O = \frac{t + c}{2} - F \) with price-matching and \( \pi^*_O = \frac{(3t+c)^2}{16t} - F \) without. Recalling that \( p^m_O = t + c \) and \( 1 - x^m = \frac{1}{2} \), the change in profit in response to a one-dollar increase in \( c \) under price-matching is given by

\[
\frac{d\pi^m_O}{dc} = \left(1 - x^m\right)\frac{\partial p^m_O}{\partial c} + p^m_O \left(-\frac{\partial x^m}{\partial c}\right) = \frac{1}{2}.
\]

Given \( p^*_O = \frac{3t+c}{2} \) and \( 1 - x^* = 1 - \frac{5t-c}{8t} \), firm O’s investment incentive without price-matching is characterized by

\[
\frac{d\pi^*_O}{dc} = \left(1 - x^*\right)\frac{\partial p^*_O}{\partial c} + p^*_O \left(-\frac{\partial x^*}{\partial c}\right) = \frac{3t+c}{8t}.
\]

Because \( \frac{3t+c}{8t} > \frac{1}{2} \) under Assumption 1, firm O’s incentive to invest in cost-reduction is weakened with a PMG, like in the case of firm B. Though we find a similar result for the low-cost firm, the reason is not exactly symmetric. Again PMGs make the extensive margin effect null. Since the

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\(^{19}\)Recall that for simplicity in analysis, we opted to include the single cost parameter \( c \) to reflect the asymmetry in cost between the two firms instead of two separate cost parameters. In that sense, firm O raising \( c \) is equivalent to firm O investing in its cost reduction.
low-cost firm is the price-leader, there is still a positive intensive margin effect for the low-cost firm from her cost reduction. However, the intensive margin effect with price-matching is smaller than that without price-matching \( \frac{3}{2} (\frac{3}{8} + \frac{c}{2t}) < \frac{1}{2} \) under Assumption 1. Moreover, in the absence of PMGs, the extensive margin effect comes back. Therefore, the net effect is stronger without price-matching.

**Proposition 3.** Under Assumption 1, price-matching by the high-cost firm weakens the low-cost firm’s incentive to increase its cost advantage.

Again, we need to check whether the low-cost firm O has an incentive to reduce the cost advantage so as to eliminate firm B’s incentive to price-match. For \( c = t + 3\alpha - \epsilon \) with \( \epsilon > 0 \), we compare (2) with (5) and find \( \pi^*_O < \pi^m_O \), which implies no incentive for such a move.

## 5 The incentive to invest in upgrading quality

### 5.1 Incentive for firm B

We proceed to analyze firm B’s incentive to lower its quality inferiority. Given \( x^m(\alpha) = \frac{t-\alpha}{2t} \) and \( p^m_B(\alpha) = t + c - \alpha \), we have \( \pi^m_B(\alpha) = \frac{(t-\alpha)^2}{2t} \). The marginal change in B’s profit in response to a marginal change in \( \alpha \) can be decomposed as follows:

\[
-d\pi^m_B(\alpha) = -\frac{\partial p^m_B(\alpha)}{\partial \alpha} x^m(\alpha) + (p^m_B(\alpha) - c) (-x''^m(\alpha)) = \frac{t - \alpha}{t}.
\]

(12)

Different from the case with \( \alpha = 0 \), now firm B’s market share under price-matching depends on the quality difference parameter \( \alpha \). As a result, the marginal effect of a quality-enhancing investment of firm B on its market share is no longer zero. The less inferior (or more superior) quality of firm B’s product expands firm B’s market share, precisely \( -x''^m(\alpha) = \frac{1}{2t} \). Since the mark-up is \( p^m_B(\alpha) - c = t - \alpha \), the total extensive margin effect is equal to \( \frac{t-\alpha}{2t} \). In addition, because \( p^m_B(\alpha) = t + c - \alpha \), the entire variation in \( \alpha \) passes through the equilibrium price, i.e.,
\[-\frac{\partial p^m_B(\alpha)}{\partial \alpha} = 1.\] Thus, the intensive margin effect is equal to $\frac{t-\alpha}{2t}$ as well. Hence, the total innovation incentives measured by the marginal profit with respect to $\alpha$, which is equal to $\frac{t-\alpha}{t}$.

Without a PMG, firm B’s investment incentive is measured by $-\frac{d\pi^*_B(\alpha)}{d\alpha}$. Recall that $p^*_B(\alpha) = \frac{5t+c-\alpha}{4}$ and $x^*_B(\alpha) = \frac{5t-c-\alpha}{8t}$.

\[-\frac{d\pi^*_B(\alpha)}{d\alpha} = -\frac{\partial p^*_B(\alpha)}{\partial \alpha}x^*_B(\alpha) + (p^*_B(\alpha) - c)(-x^*_B(\alpha)) = \frac{5t-c-\alpha}{16t}. \quad (13)\]

The extensive margin effect and the intensive margin effect are given by the same magnitude of $\frac{5t-c-\alpha}{32t}$, which leads to the total effect of $\frac{5t-c-\alpha}{16t}$. If we compare the total effect under the price-match with that without it, we can see that the incentive to upgrade quality is weaker with price-matching if and only if

\[c < 15\alpha - 11t. \quad (14)\]

Why does the size of cost difference matter? Under PMGs, the investment incentive is independent of $c$ because the market share and the mark-up for the high-cost firm are independent of $c$. By contrast, the intensive margin effect and extensive margin effect both increase with $c$ when firm O’s quality improves without PMGs. In other words, the investment incentives for quality are stronger when its cost disadvantage is bigger, which is intuitive. As a result, the high-cost firm’s investment incentive is smaller under PMGs for a small cost disadvantage, while the opposite is the case for a large $c$. Note that for $c > t + 3\alpha$, there can be such a $c$ if $t + 3\alpha < 15\alpha - 11t$, which is equal to $t < \alpha$.

**Proposition 4.** Under Assumption 1, price-matching weakens the brick-and-mortar firm’s incentive to invest in quality-improvement for a sufficiently small $c < 15\alpha - 11t$, but strengthens it otherwise.

**5.2 Incentive for firm O**

We can examine firm O’s incentive to improve quality in a similar manner. First, consider the price-matching situation. Recall $\pi^m_O = p^m_O(1-x^m) = (t+c-\alpha)\frac{t+\alpha}{2t}$. Hence,
\[
\frac{d\pi^m_O}{d\alpha} = \frac{\partial p^m_O}{\partial \alpha} (1 - x^m) + \left(- \frac{\partial x^m}{\partial \alpha}\right) p^m_O = (-1) \frac{t + \alpha}{2t} + \left(\frac{1}{2t}\right) (t + c - \alpha) = \frac{c - 2\alpha}{2t} \quad (15)
\]

Again, the market share is independent of \(c\) under PMGs. The intensive margin effect is negative because the high-cost firm’s price-match leads to an even lower mark-up when the quality difference increases.

Now consider the no price-matching case. Recall \(\pi^*_O = p^*_O (1 - x^*) = \left(\frac{3t + c + \alpha}{2}\right) \left(\frac{3t + c + \alpha}{8t}\right)\).

Similarly, we have

\[
\frac{d\pi^*_O}{d\alpha} = \frac{\partial p^*_O}{\partial \alpha} (1 - x^*) + \frac{\partial (1 - x^*)}{\partial \alpha} p^*_O = \frac{1}{2} \left(\frac{3t + c + \alpha}{8t}\right) + \frac{1}{8t} \left(\frac{3t + c + \alpha}{2}\right) = \frac{3t + c + \alpha}{8t} \quad (16)
\]

Without PMGs, the intensive margin effect turns positive. The incentive to upgrade quality for firm O is weaker with price-matching compared to no price-matching if and only if \(t + 3\alpha > c\), which always holds under Assumption 1.

**Proposition 5.** Under Assumption 1, when it adopts price-matching, the low-cost firm has lower incentives to increase its product’s quality superiority.

### 6 Discussion

In this section, we will consider a short extension to production inefficiency. We also discuss “showrooming” and “hassle costs” in light of our results, two topics that have been analyzed in other papers in the price-matching literature.
6.1 Production Inefficiency

It is well known that there is an ingrained inefficiency in the Hotelling setup – the high-cost firm serves too large a share of the demand relative to the social optimum. The reason why the low-cost firm sells too few units in the Hotelling model is that when products are differentiated, the cost advantage does not lead to a market share increase at full strength. In other words, consumers’ relative preferences for different products, beyond a simple price comparison, causes frictions from a social efficiency perspective. These frictions in consumers’ switching prevent the low-cost producer from securing a socially efficient market share.

Let us first confirm this point in our model. Note that the social optimum requires marginal-cost pricing and hence $\bar{\tilde{p}}_O = 0$ and $\bar{\tilde{p}}_B = c$ in our model. With marginal-cost pricing, firm B’s socially optimal market share is computed as $\bar{x}(c, 0) = \frac{t-c-\alpha}{2t}$. Under Assumption 1, firm B prefers price-matching and its market share is $x^m = \frac{t-\alpha}{2t}$. Hence, the socially optimal market share for firm B is less than firm B’s market share with price-matching when $\frac{t-c-\alpha}{2t} < \frac{t-\alpha}{2t}$, which holds for any $c > 0$. With PMGs, the offline (high-cost) firm would, by covering half of the market, serve an even larger share of the demand relative to the social optimum.

From a policy point of view, one concerning situation arises when the high-cost firm exits the market due to its sufficiently large cost disadvantage so that the market becomes a local monopoly by the low-cost firm. In fact, many small towns experience the closure of local retail shops and malls due to intense competition from online rivals. In this situation, consumers may have no choice other than to buy at least certain goods from online firms despite their strong preferences toward offline shopping (e.g., some elderly do not know how to shop online; they are close to $x = 0$). In this sense, this production inefficiency from PMGs does not necessarily mean that consumer welfare would be higher without PMGs because the analysis in this paper focuses on the interior solutions where both firms are active.

20See Lesson 3.4 on pg. 54 of Belleflamme and Peitz (2015).
6.2 Showrooming and Hassle Costs

Given the scope of our paper, we do not consider the possibility of brick-and-mortars using price-matching to prevent the practice of “show-rooming” on which several papers focus (e.g. Liu (2013); Wu et al. (2015); Mehra et al. (2017); Wang and Wright (forthcoming)). Also, the presence of “hassle costs” may limit the anti-competitive aspect of price-matching. If the burden of proving the existence of lower prices elsewhere fall on the buyer, this usually entails him a) searching for a lower price, b) bringing proof in the form of a flyer, printout or display of an advertisement on a smartphone, etc., c) potentially filling out paperwork (some stores require registration) d) waiting for a store employee to process the claim, and so on. Even after incurring such costs, the buyer may not always get the price-match due to failure to comply with all the conditions or due to inconsistent implementation of such policies by store employees. Hviid and Shaffer (1999) show that the presence of even small hassle costs can diminish, if not eliminate, the collusive implications of price-match policies.

The caveat of the argument for hassle costs is that it holds when the burden of price-matching is mostly on the customer. If however, firms undertake the responsibility of matching their rivals’ prices and keep their posted prices equal to those of rivals, consumers face no hassle costs. Although we often observe both price dispersion and buyers bringing in rivals’ advertisements to get price-matches, which is the criticism by Corts (1997) of non-hassle-costs based models of price-matching, there is a recent trend where the sellers are trying to minimize the hassle of price-matching. This may be because the sellers are becoming aware of how hassle costs limit the effectiveness of PMGs. For instance, in 2014, Walmart announced a smartphone app that will search for lower prices elsewhere and issue refunds automatically as long as customers upload their Walmart receipts on to the app.21 This is more prevalent in the U.K. where Asda, Tesco and Sainsbury’s offer similar PMGs in which the burden of finding lower prices and refunding the buyer falls on the seller.

21Source: “Walmart To Competitors: Catch Our Savings If You Can.” Retrieved from https://corporate.walmart.com/_news_/news-archive/2014/06/05/walmart-to-competitors-catch-our-savings-if-you-can
Price-matching as a price discrimination device is an alternative argument that seeks to explain why firms price-match (see Png and Hirshleifer (1987) and Corts (1997) among others). When a price-matching firm has posted a price higher than the rivals’ prices, only a small percentage of customers tend to exercise PMGs. This is not surprising if hassle costs are high, and the burden on obtaining the price-match is on the buyer. At the same time, the potentially heterogeneous hassle costs play a role of self-selectively separating buyers into two types: high hassle-cost consumers who do not bother price-matching and low hassle-cost consumers who do. Hence, the firm manages to charge the regular high price to the high-type and the discounted price after price-matching to the low-type. Because in our model we consider the case where posted prices are already equal due to a PMG, studying the possibility of and implications for price discrimination is beyond the scope of your paper.22

7 Conclusion

Economists have studied various incentives for and effects of PMGs and similar business practices. However, to our knowledge, the potential impact on investment incentives associated with price-matching has not been given sufficient attention. The effects of PMGs on investment incentives of the imposing firm or the competing rivals should be one of the key issues for policymakers since innovation and investment fundamentally determine the welfare of consumers and society. Our study is particularly relevant in the current business environment where online retailers such as Amazon have rapidly expanded, and many competing brick-and-mortar retailers have responded with price-matching policies.

We study how PMGs have implications for investment incentives regarding cost reduction and quality enhancement, using a sequential price duopoly competition game in the Hotelling model. We first show that PMGs can dampen the investment incentives for cost reduction by both com-

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22 We admit that PMGs are likely to influence how long consumers keep searching for a lower price and also their reservation prices, which in turn can affect optimal pricing by firms. For example, Janssen and Parakhonyak (2013) and Yankelevich and Vaughan (2016) study the implications of PMGs on consumer search.
peting firms. This result arises because the low-cost price leader does not want to invest much, knowing that gaining more market share is limited by the PMGs, and the mark-up improvement from a cost reduction is also smaller with PMGs. The high-cost price follower also wants to invest less because PMGs are an alternative way to expand the market share, and the potential mark-up improvement passes through to the consumers via the price.

We also show that PMGs unconditionally weaken the low-cost firm’s investment incentive to improve quality. A marginal increase in the quality difference does not lead to a higher market share under PMGs. Instead, it increases the market asymmetry and thus lowers the equilibrium price and its profit. Also, the high-cost firm’s investment incentive to reduce the quality gap depends on the cost difference between the two firms. The reason is that reducing a quality difference becomes more important when the cost difference is more significant.

While we admit that our theoretical results are derived under specific assumptions, we believe that the results have important policy implications for prevailing PMGs. The paper highlights the potential social costs associated with PMGs by brick-and-mortar stores competing against lower-cost online rivals. They may hurt consumers in the static sense and adversely affect the firms’ incentives to reduce their costs and improve the quality of product and service. Our results are particularly relevant when PMGs are effectively giving back the market share to the high-cost firm, and when quality and price are close substitutes in determining the market shares.
References


Liu, Q. (2013). ’showrooming’and price-matching guarantee. *Available at SSRN 1862181*.


Appendix: Static Effects of Price-Matching

A.1 Effect on online rival

The impact on firm O can be examined by studying how firm O’s payoff changes with the price-match compared to that without it. First of all, let us recall the standard result, which is obtained in our model as well, that a price-match commitment can soften price competition. Under Assumption 1, price-matching increases equilibrium prices relative to no price-match prices:

$$p^m_O - p^*_O = \frac{1}{2} (c - t) > 0; \quad p^m_B - p^*_B = \frac{1}{4} (c - t) > 0.$$ 

However, the higher prices do not necessarily imply that the online seller also benefits from weakened competition. The comparison between firm O’s payoff with the price-match and without it can be decomposed into two opposing forces:

$$\pi^m_O - \pi^*_O = (p^m_O - p^*_O) Q^m_O + p^*_O (Q^m_O - Q^*_O)$$

$$= \frac{(c - t)}{4} - \frac{(c + 3t)(c - t)}{16t} = -\frac{(c - t)^2}{16t} < 0$$

On the one hand, firm B’s price-match weakens price competition, which makes firm O enjoy a higher per-unit markup on existing sales, captured by the first term on the right-hand side of the equation above. On the other hand, firm O loses market share due to the PMG, represented by the second term above. We find that price-matching makes the online seller earn less profit because the negative market-share effect outweighs the positive mark-up effect. Graphically, it is clear from Figure 1 that firm B’s isoprofit curve passing through point M is to the left of the one passing through point E.
A.2 Effect on consumers

Antitrust concerns are usually focused more on the impact on consumers than on rivals. We showed that equilibrium prices increase due to price-matching. Does this necessarily imply that all consumers will be worse off? To answer this question, let us start by noticing that there are three groups of consumers who are affected by a PMG differently based on their location. First, consumers located close to firm B in \( x \in \left[0, \frac{5}{8} - \frac{c}{8t}\right) \) buy from firm B regardless of price-matching. These buyers are worse off because \( p^m_B > p^*_B \). Similarly, the consumers located in \( x \in \left[\frac{1}{2}, 1\right] \) always buy from firm O and they also become worse off because \( p^m_O - p^*_O > 0 \). In contrast, consumers in the region \( x \in \left[\frac{5}{8} - \frac{c}{8t}, \frac{1}{2}\right) \) would have bought from firm O without the price-match but switch to firm B in the presence of the price-match. Hence, the net surplus of a consumer \( x \) in this group is computed as

\[
V^*(p^*_O, x) = v - t(1 - x) - \left(\frac{3t + c}{2}\right)
\]

with no PMG and

\[
V^m(p^m_B, x) = v - tx - (t + c)
\]

with PMGs. The net change in consumer surplus denoted by \( \Delta V(x) \) is given by

\[
\Delta V(x) \equiv V^m(p^m_B, x) - V^*(p^*_O, x) = \frac{-1}{2}(c - t) + \frac{t(1 - 2x)}{2}
\]

(17)

A consumer switching due to the price-match ends up paying the higher price \( p^m_B = t + c \) compared to \( p^*_O = \frac{3t + c}{2} \) without the price-match. The first term in (17) captures this negative price effect. However, this switching consumer now saves on “travel costs” by choosing a more preferred mode of shopping given equal prices by both firms. The second term measures this positive preference effect. Note that \( \Delta V(x) \) decreases with \( x \) because the price effect is independent of \( x \), but the preference effect decreases with \( x \). The largest preference effect occurs for the consumer located at \( x^* = \frac{5}{8} - \frac{c}{8t} \) (and there is no preference effect for \( x = 1/2 \)). But we can verify that even this consumer finds the price-match undesirable, i.e.,

\[
\Delta V \left(\frac{5}{8} - \frac{c}{8t}\right) = -\frac{1}{4}(c - t) < 0
\]

which ensures \( \Delta V(x) < 0 \) for every switching consumer. That is, \( \Delta V(x) < 0 \) for \( \forall x \in \left[\frac{5}{8} - \frac{c}{8t}, \frac{1}{2}\right) \).

\[23\] For example, if a buyer can get the lowest price at the store closest to him by showing proof of a lower price at a store farther out from his home, this saves him the trouble of a longer trip. Similarly, if he finds something cheaper online but does not want to wait for the item to arrive, he can immediately go to the local brick-and-mortar store that price-matches and get the item at the lower online price.
The total consumer surplus denoted by $S$ is computed from aggregating all consumers’ net surplus\(^{24}\). With the price-match, it is given by

$$S^m = \int_0^{1/2} (v - tx - p_B^m)dx + \int_{1/2}^1 (v - t(1 - x) - p_O^m)dx = v - \frac{5}{4}t - c.$$ 

Without the price-match, the aggregate consumer surplus is

$$S^* = \int_0^{x^*} (v - tx - p_B^*)dx + \int_{x^*}^1 (v - t(1 - x) - p_O^*)dx = v - \frac{103}{64}t - \frac{21}{32}c + \frac{c^2}{64t}$$

where $x^* = \frac{5}{8} - \frac{c}{8t}$. The difference in aggregate consumer surplus due to the price-match is

$$\Delta S = S^m - S^* = -\frac{(c - t)(c + 23t)}{64t} < 0$$

which verifies that price-matching leads to a decrease in consumer surplus as every consumer is worse off from the higher prices due to the price-match policy and she is not sufficiently compensated for the loss by a positive preference effect.

### A.3 Effect on social welfare

Since total welfare is the sum of firm profits and consumer surplus, the welfare effects of price-matching are consistent with what we have described regarding firm profits and consumer surplus. One notable point is how the change in firm B’s profit due to the price-match is substantial relative to the impact on its rivals and consumers.

Using prior results, we can obtain the welfare with and without price-match as follows: $W^m = \pi_B^m + \pi_O^m + CS^m$ and $W^* = \pi_B^* + \pi_O^* + CS^*$ from which the net change in welfare is given by

$$\Delta W \equiv W^m - W^* = -\frac{(c - t)(7c + t)}{64t} < 0.$$ \hspace{1cm} (18)

\(^{24}\)Note that we use $S$ for consumer surplus and later use $W$ for social welfare.
The offline seller’s gain is not enough to offset the loss to consumers and the decrease in online rival’s profits.

B  Appendix: The simultaneous pricing game

Suppose that firm B did not choose to price-match. Then, generally we can think of an alternative version of price competition that the two firms compete with their simultaneous price choices. For this simultaneous pricing game, we use the hat symbol (ˆ) for each variable to make them distinct from the variables in the sequential pricing game where we do not use the hat symbol.

The analysis when there was no price-match by firm B is straightforward. As usual, the analysis starts with characterizing the demand for each firm by identifying the indifferent consumer’s location \( \hat{x} = \frac{1}{2} + \frac{\hat{p}_O - \hat{p}_B}{2t} \), which gives the demand for B, \( \hat{Q}_B(\hat{p}_B, \hat{p}_O) = \hat{x} \), and for O, \( \hat{Q}_O(\hat{p}_B, \hat{p}_O) = 1 - \hat{x} \). Then, we set up each firm’s profit maximization problem. For firm B, it is set up as \( \max \hat{\pi}_B = (\hat{p}_B - c) \hat{Q}_B \) and for firm O as \( \max \hat{\pi}_O = \hat{p}_O \hat{Q}_O \). Deriving the first-order necessary conditions for their prices, we obtain following two best responses: \( \hat{p}_B(\hat{p}_O) = \frac{1}{2} (t + \hat{p}_O + c) \) and \( \hat{p}_O(\hat{p}_B) = \frac{1}{2} (t + \hat{p}_B) \). Solving these two equations as a system, the prices in Nash equilibrium are derived as

\[
\hat{p}_B^* = t + \frac{2}{3} c; \quad \hat{p}_O^* = t + \frac{1}{3} c. \tag{19}
\]

where the asterisk at the subscript (\( ^* \)) signifies that we are considering the standard price competition without a price-match. Substituting (19) into \( \hat{x}(\hat{p}_B, \hat{p}_O) \), the demands in equilibrium are derived as

\[
\hat{Q}_B = \hat{x}^* = \frac{1}{2} - \frac{c}{6t}; \quad \hat{Q}_O = 1 - \hat{x}^* = \frac{1}{2} + \frac{c}{6t}. \tag{20}
\]

As firm B’s marginal cost is higher than that of firm O, the demand for firm B is smaller than that of firm O. The demand gap \( \frac{\hat{Q}_B}{\hat{Q}_O} \) increases with \( c \), but decreases with \( t \). Using the derived prices and demands, the equilibrium profits are equal to

\[
\hat{\pi}_B^* = (\hat{p}_B^* - c) \hat{x}^* = \frac{(3t - c)^2}{18t}; \quad \hat{\pi}_O^* = \hat{p}_O^* (1 - \hat{x}^*) = \frac{(3t + c)^2}{18t}. \tag{21}
\]
For an interior solution \( \hat{x}^* \in (0, 1) \), we need \( c < 3t \) without which the market equilibrium is characterized by the so-called tipping equilibrium such that the online seller monopolizes the market. Intuitively, this corner solution arises when the cost disadvantage \( c \) is sufficiently large that firm B cannot survive as an effective competitor vis-à-vis the more efficient online rival. Aforementioned, we assume a full market coverage. The net surplus of the indifferent consumer must be non-negative, i.e. \( v - t \hat{x}^* - \hat{p}_B^* \geq 0 \) where \( v \) denotes the intrinsic value of a product. This condition leads to \( v \geq \frac{3}{2} t + \frac{1}{2} c \). Intuitively, the market will be fully covered once the intrinsic value is large enough.

Now consider the other subgame ensuing from the brick-and-mortar firm’s price-match commitment. Under the price-match announcement, one candidate for the equilibrium would be that the market will be evenly split as long as both firms are active and they charge the same price. In this case the marginal consumer is located at \( \hat{x}^* = 1/2 \) and both firms charge the price such that they extract the entire surplus from the marginal consumer:

\[
\hat{p}_O^m = \hat{p}_B^m = v - \frac{t}{2},
\]

where the superscript \( m \) indicates that currently we consider the price-match. The non-negative price constraint requires \( v \geq t/2 \). With this equal price, both firms earn the profits of

\[
\hat{\pi}_O^m = \frac{1}{2} (v - \frac{t}{2}); \quad \hat{\pi}_B^m = \frac{1}{2} \left( v - \frac{t}{2} - c \right)
\]

where firm B will be active only if \( v \geq t/2 + c \). Both of the constraints on price and no-exclusion of firm B at the matched price are subsumed by the full coverage assumption \( v \geq \frac{3}{2} t + \frac{1}{2} c \).

However, before we can assert that (23) represent the price-match subgame payoffs, we need to check whether \( \hat{p}_O^m = \hat{p}_B^m = v - \frac{t}{2} \) is sustained as an equilibrium pricing strategy. In fact, this is not an equilibrium. To see this point, consider firm B’s deviation to \( \hat{p}_B^d = v - t \). Then, consumers see no need to claim the price match because now \( \hat{p}_B^d = v - \frac{t}{2} > \hat{p}_O^d = v - t \). Firm B’s new payoff
is \( \tilde{\pi}_B^d = v - t - c \) and this deviation strategy is profitable if

\[
v - t - c > \frac{1}{2}(v - \frac{t}{2} - c),
\]

that is, \( v > c + \frac{3t}{2} \). This condition has no conflict with any restriction to the equilibrium and basically it boils down to \( v \) being sufficiently high, which is consistent with the full coverage assumption. This implies that there will be no equilibrium with simultaneous pricing such that both firms have a positive market share with the full market covered.