Double Marginalization and Misplacement in Online Advertising*

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Abstract

Internet users often surf multiple websites as a bundle to fulfill their needs and "pay" for the content by watching ads. We study how such complementary online businesses choose advertising policies. Two forces distort the equilibrium away from the industry optimum and the efficient outcome. First, websites place too many ads (double marginalization). Second, given the total advertising volume at equilibrium, websites misallocate ads across themselves (misplacement). Competition in one market segment may eliminate double marginalization but exacerbate misplacement. Introducing micropayments removes misplacement, but the welfare consequences are ambiguous. Policymakers thus need caution in applying the standard remedies to zero-price markets. (JEL codes: D21, D40, L23, L42, L86)

Keywords: Platforms, Advertising, Misplacement, Double Marginalization, Competition

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1 Introduction

Internet users often surf multiple websites (or mobile apps) to fulfill their needs. Some websites play the role of platforms (e.g., search engines, social media platforms, news aggregators), which link their users to content producers (e.g., news media, blogs, product review sites). For example, consider a Google, Facebook, or Twitter user who scrolls through either platform and then clicks on a link to a news story of interest. As would typically be the case in this example, many websites do not directly charge users; instead, they "tax" users' attention via advertising. The ads impose nuisance costs on the users and thus may reduce the demand for the service and possibly decrease ad revenue. In this environment we study the incentives of websites and the platform to choose their advertising policies.

The contribution of this paper is three-fold. First, we identify two forces that render any equilibrium suboptimal in terms of industry profit and total welfare. The first force is the standard *double marginalization*: Individual websites do not take into account the adverse effects their ad placement has on other sites, specifically, the decrease in user visits to those sites through the platform. As a result, the platform and websites place more ads than the industry-optimum or the efficient level. The second force is the *misplacement* of ads, i.e., the platfrom and websites could potentially improve industry profits by reallocating advertisements from one site to another relative to the equilibrium allocation, while maintaining the consumer's disutility at the equilibrium level. For example, suppose that the platform is more effective than websites at converting ads into revenue–e.g., the platform's ads impose lower disutility on consumers, or its better targeting ability ensures a higher click-through rate.[1] In such a case, any equilibrium entails misplacement, as the joint profit would increase if the platform placed more ads and websites reduced their advertising volumes, without changing the total volume of advertising. While double marginalization is well known, this misplacement is unique to our model, in which websites are heterogeneous.

Second, we demonstrate a trade-off where standard solutions to the double marginalization problem, such as fostering competition among certain sites, can aggravate the misplacement

[1]Heterogeneous nuisance costs can be equivalently modeled as heterogeneous ads revenue per visit of a user. The literature on online advertising has supported the heterogeneous technology among advertisers in converting consumer attention to ads revenues (e.g., Evans (2008, 2009); Goldfarb and Tucker (2011); Athey et al. (2018)).

problem. The standard argument on the pricing of complements suggests that competition mitigates double marginalization.[2] Our model confirms this intuition: Competition between websites mitigates double marginalization and benefits consumers. At the same time, however, competition may exacerbate misplacement and decrease the industry profit. For example, suppose the equilibrium without competition entails misplacement, where websites place too few ads relative to the platform. If websites face competition, they further decrease the advertising volume, which in turn incentivizes the platfrom to place more ads. Standard remedies for traditional offline markets may have an unintended consequence for online advertising markets where attention replaces payments.

Third, we show that misplacement stems from the lack of monetary instruments in online attention economy, which contributes to the understanding of zero-price markets.[3] Specifically, we augment the model by allowing the platform and each website to place ads and charge consumers via per-visit monetary transfers, referred to as "micropayments." The equilibrium in such a game still entails double marginalization, but it no longer has misplacement. We then examine how the introduction of micropayments affects industry profit and consumer surplus. When all websites are highly effective at converting ads into revenue, the introduction of micropayments benefits consumers: in the new equilibrium, websites place more ads but reward consumers for watching ads through monetary transfers. In other cases, the introduction of micropayments may harm consumers and benefit only websites by enabling them to charge consumers for the content and thus extract surplus.

While we motivate the model as one in which the platform and websites place ads, we can alternatively interpret the strategy of a website as the amount of personal data it collects from visitors. For example, collecting personal data imposes disutility on visitors, and websites may differ in their ability to monetize data. Thus our model also speaks to coordination problems of online businesses that request access to data from visitors. In sum, we argue

[2]For example, two monopolists that sell complementary goods—such as tea and sugar— will face double marginalization. If a new firm enters and intensifies competition in the tea market, it will mitigate double marginalization and increase the industry profit (Rey and Tirole, 1986; Shleifer and Vishny, 1993; Lerner and Tirole, 2004, 2015; Dellarocas, 2012).

[3]Jullien and Sand-Zantman (2018) also discuss that the combination of a "missing price" for content and site heterogeneity generate the externality problem, which ends up comprising efficiency in the context of the sponsored data and net neutrality debate.

that complementary actors in the attention economy, such as websites and browsers, may fail to coordinate their strategies to place ads or collect data, leading to both classical and novel market distortions—i.e., double marginalization and misplacement. Competition or monetary instruments play different roles in correcting or exacerbating such distortions.

The rest of this paper is organized as follows. After discussing related literature, we present the baseline model in Section 2. This section also discusses modeling assumptions and offers an alternative interpretation of the model from the perspective of personal data collection. Section 3 characterizes the equilibrium and identifies two forces that render the equilibrium suboptimal in terms of industry profit and social welfare. We then analyze the impacts of competition in Section 4. Section 5 incoporates micropayments and shows that micropayments eliminate misplacement. We then examine how the introduction of micropayments affect consumer surplus and industry profit. Section 6 discusses some implications of our result, and Section 7 concludes. We relegate all proofs in Appendix A.

Related Literature Double marginalization, first pointed out by Cournot (1838), has been extensively studied in the context of complementary goods (e.g., Spengler (1950); Rey and Tirole (1986); Shleifer and Vishny (1993); Lerner and Tirole (2004, 2015)). In contrast, to the best of our knowledge, misplacement, which leads to our central trade-off, has not been discussed in the extant literature, especially in the setup where multiple firms can demand attention from consumers trying to accomplish a single task. The closest analog to misplacement of which we are aware arises in Schwartz (1989), where imperfectly competing sellers of substitutes have asymmetric marginal costs. Dellarocas's (2012) model resembles ours in that it also studies double marginalization in online advertising. However, our paper differs from his model in that he studies the double markup problem in product pricing, whereas we focus on the interaction between ads misplacement and the double marginalization in the amount of advertisement.

A strand of literature studies "vertical cooperative advertising." It stems from Berger (1972), continues through Cao and Ke (2019), and is surveyed by Jørgensen and Zaccour (2014). There, coordination problems arise between manufacturers and retailers, each of which may place ads to increase demand. In contrast, our paper focuses on obstacles faced by multiple websites (or, more broadly, ad-funded platforms) in coordinating policies determining how much advertisement to show.

Another strand of literature studies situations in which platforms, such as search engines, direct users to sellers (e.g., Athey and Ellison (2011); Hagiu and Jullien (2011, 2014); Eliaz and Spiegler (2011); White (2013); Gomes (2014); Burguet et al. (2015); de Cornière (2016)). These models do not have the misplacement that we study, because the sellers can make monetary transfers to the platform. Our model relates to de Cornière and Taylor (2014), because in both models users surf from an platform to content producers, which cannot make monetary transfers to one another. Suppose that multiple content producers compete in one category of content. Then, if we introduce horizontal differentiation across websites, the platform may be biased against the websites that display many ads. de Cornière and Taylor (2014) study such a recommendation bias in selecting one group of publishers rather than the other.[4] In their terminology, we abstract away from the recommendation bias and study the lack of coordination between the search engine and publishers, particularly the roles of competition and payments.

2 Model

Consumers visit a platform—such as Google or Facebook— that directs visitors to a relevant website, such as a news site or blog. The platform and websites choose their advertising volumes in order to maximize their advertising revenues. A consumer's decision of whether to visit the platform depends on the value of the content and the expected disutilities from advertising.

Formally, the model consists of a unit mass of consumers, $n \in \mathbb{N}$ websites, and a platform (p). Let $W := \{1, ..., n\}$ denote the set of the websites and $\overline{W} := W \cup \{p\}$ denote the set consisting of all websites and the platform. We use w for a generic element of W and i for a generic element of $\overline{W}$. When we do not distinguish between the platform and a website, we refer to a generic player $i \in \overline{W}$ as business i.

The game unfolds as follows: First, each business $i \in \overline{W}$ simultaneously chooses the

[4] Similarly, Burguet, Caminal, and Ellman (2015) consider the situation in which search ads and display ads are substitutes and study the search engine's incentives to distort search results.

advertising volume, $a_i \geq 0$. Then for each consumer, exactly one website is selected by Nature as "relevant." Each website is ex-ante equally likely to be relevant for each consumer, so ex-post, each website w becomes relevant to mass $\frac{1}{n}$ of consumers. Prior to visiting the platform, each consumer observes the average advertising volume across the websites, $\frac{1}{n}\sum_{i\in W} a_w$, and the advertising volume a_p chosen by the platform.[5] Each consumer also knows her value v of the relevant website but does not know which website is relevant. Then each consumer decides whether or not to visit the platform. A consumer who visits the platform is automatically directed to the website that is relevant to her; the assumption that consumers are always directed to relevant websites (regardless of their advertising volumes) is without loss of generality as we discuss in the next subsection.

The value v of the relevant website is distributed across consumers according to distribution function F that has a positive density on its support $[0, \overline{v}]$ and an increasing hazard rate $\frac{f}{1-F}$. As a result, consumers are heterogeneous, whereas websites are homogeneous in the sense that the value for a consumer of visiting website w depends only on whether the website is relevant and not on its identity w.

The payoff to each consumer from not visiting the platform is normalized to 0. If a consumer visits the platform—which directs her to the relevant website—the consumer enjoys the content but incurs disutilities from advertising. In this case, her ex-post payoff is $v - \delta_p(a_p) - \delta_w(a_w)$. Here, v is the consumer's value of her relevant content, and $\delta_p(a_p)$ and $\delta_w(a_w)$ are disutilities from advertising placed on the platform and relevant website w, respectively. For each $i \in \overline{W}$, function $\delta_i : \mathbb{R}_+ \to \mathbb{R}_+$ maps advertising volume a_i to disutility imposed on visitors. Recall that when a consumer decides whether to visit the platform, she does not know which website is relevant but knows the distribution of advertising volumes and understands how the platform directs users to websites. As a result, a consumer bases her decision of whether to visit the platform on $v - \delta_p(a_p) - \frac{1}{n}\sum_{w\in W}\delta_w(a_w)$.

The payoff of each business is its advertising revenue, defined as its advertising volume a_i multiplied by the mass of consumers who visit the business. Thus if mass m of consumers visit

[5] We could alternatively assume that each consumer observes the profile $(a_i)_{i\in\overline{W}}$ of advertising volumes for all businesses. An advantage of the current assumption is that the equilibrium remains the same even if we allow consumers to decide whether to visit their relevant websites after visiting the platform. See the discussion in Section 2.1.

the platform, then the platform and website w earn payoffs $a_p m$ and $\frac{1}{n} a_w m$, respectively.

Our solution concept (hereafter, "equilibrium") is pure-strategy subgame perfect equilibrium (SPE) in which a positive mass of consumers visit the platform. This restriction excludes a trivial SPE in which every business sets a large a_i and no consumer visits the platform. To facilitate the analysis, we restrict the functional form of the disutility functions as follows:

Assumption 1. For each $i \in \overline{W}$, $\delta_i(a) = \gamma_i a^k$ for some $\gamma_i > 0$ and $k > 1$.

Under Assumption 1, the disutility functions of businesses differ only in parameter γ_i. We can interpret a lower γ_i as an advertising technology that is more effective or less intrusive towards consumers. Assumption 1 is equivalent to δ_i being a homogeneous function. This assumption ensures that $a\delta_i'(a)$ is proportional to $\delta_i(a)$, which enables us to characterize the total disutility levels for the equilibrium and the industry optimum by using the first-order conditions.

2.1 Discussion of Modeling Assumptions

Heterogeneous websites and platform. One of our key insights (i.e., misplacement) is relevant when consumers face a higher marginal disutility from advertising on one business than the other, i.e., $\gamma_i > \gamma_j$ for some i and j. For example, suppose that a consumer visits a social media website (i.e., the platform) and then clicks a link to visit a news website (i.e., a website). Suppose the former uses display ads and the latter uses video ads. If consumers find video ads that appear prior to news videos more annoying than display ads, they face higher $\delta_w'(a)$ than $\delta_p'(a)$.[6] Generally, such a difference between $\delta_p(\cdot)$ and $\delta_w(\cdot)$ would arise if websites employ different advertising modes, and consumers find one mode more annoying than the other. The model assumes that each business earns the same revenue per unit of advertisement. This assumption is without loss of generality in the following sense: Suppose that by choosing advertising volume b_i, business i earns a revenue of $r_i b_i$ per visit with $r_i > 0$ and imposes disutility $\hat{\delta}_i(b_i)$ on visitors. We can redefine the website's strategy as $a_i := r_i b_i$ and disutility

[6]Academic empirical evidence on consumers' attitudes toward different advertising modes is sparse, but there is more suggestive evidence of this example from non-academic media sources, e.g., https://www.emarketer.com/content/why-consumers-avoid-ads and https://www.vieodesign.com/blog/new-data-why-people-hate-ads.

as $\delta_i(a_i) := \hat{\delta}_i(a_i/r_i)$. If $\hat{\delta}_i$ satisfies Assumption 1, so does function δ_i, and we can view a higher r_i as a lower γ_i, i.e., a higher marginal revenue r_i from placing ads translates into lower disutilities. For example, suppose that, compared with website w', website w has better access to user data, enabling advertisers to target users; consequently, it can sell advertising slots at higher prices. Website w then faces a higher marginal revenue from placing ads than website w', which is equivalent to $\delta'_w(a) < \delta'_{w'}(a)$.

Consumer's decision on the platform. We assume that once a consumer visits the platform, she is directed to the relevant website regardless of its advertising volume. This is without loss of generality in the sense that even if we allow consumers to choose whether and which website to visit on their own, we obtain the same equilibrium in the original model. To see this, note that a consumer finds it optimal to visit the platform if $v - a_p - \frac{1}{n}\sum_{w\in W} a_w \geq 0$ and finds it optimal to visit the relevant website (conditional on visiting the platform) if $v - \frac{1}{n}\sum_{w\in W} a_w \geq 0$. Because the former implies the latter, the equilibrium we study continues to be an equilibrium even if we allow consumers to decide which website to visit after visiting the platform.

Zero marginal cost. We assume that websites face zero marginal cost of serving users. We impose such an assumption for two reasons. First, the near-zero marginal cost of serving users is a feature of digital goods and online services (Rifkin, 2014). Second, while some of our results depend on the assumption of zero marginal cost, introducing a positive cost (i.e., each website earns $a_i - c_i$ per visit) increases the notational burden for other results without adding new insights.

Single-homing consumers. Unlike the multi-homing literature (e.g., Anderson, Foros, and Kind (2016) and Ambrus, Calvano, and Reisinger (2016)) in which advertisers' value for the second impression and subsequent ones is lower than for the first impression, we assume that consumers do single-home and thus there is no wasted impression over different levels of advertising. However, a single-homing in our model may be matched with different websites over their multiple surfs.

2.2 Alternative Interpretation: The Collection of Personal Data

We have described the model as one of online advertising markets where a_i captures an advertising volume. Alternatively, we can interpret the model in the context of personal data collection. For example, suppose that the platform is a mobile browser. In this example, the browser and a website are complementary components that fulfill consumers' needs to access content. Leaving aside any issues of advertisement to the primary interpretation, in this context the focus is on how each firm collects users' personal data. We can now interpret a_i as the level of data collection and $\delta_i(a_i)$ as associated disutilities, such as a potential loss from data leakage or a consumer's privacy concern. Collecting more data imposes a higher disutility on consumers but increases the revenue per consumer of the browser or a website, possibly because of better targeting.

The browser and websites may differ in the disutility they impose on consumers or in the value they extract from user data. For example, both the browser and websites may collect data in a way that purports to improve the user experience, but the browser may store this more securely than websites and thus impose less perceived privacy loss. On the revenue side, the businesses may earn differently from the same data if one business operates other data-driven services while businesses does not. In view of this interpretation, the coordination problems we study are not purely limited to situations that involve advertising but also can be seen to arise in other online settings with complementary components in which monetary transfers are not practical.

3 Equilibrium: Double Marginalization and Misplacement

We identify two forces that render equilibrium suboptimal in terms of industry profit and social welfare. First, we prepare notations. Given distribution F, define $D = 1 - F$. Given the strategies of the platform and the websites, the total disutility is defined as

$$\Delta := \delta_p(a_p) + \frac{1}{n}\sum_{w\in W}\delta_w(a_w).$$

The mass of consumers who visit the platform in equilibrium is written as $D(\Delta)$, and the payoff of the platform and that of each website w are $a_p D(\Delta)$ and $\frac{1}{n} a_w D(\Delta)$, respectively. We say that the profile of advertising volumes, $\boldsymbol{a}^{\Pi} = (a_i^{\Pi})_{i \in \overline{W}}$, is *industry-optimal* if it maximizes the joint profit given consumers' optimal behavior:

$$\boldsymbol{a}^{\Pi} \in \arg \max_{(a_i)_{i \in \overline{W}} \in \mathbb{R}_+^{n+1}} \left(a_p + \frac{1}{n} \sum_{w \in W} a_w \right) D \left(\delta_p(a_p) + \frac{1}{n} \sum_{w \in W} \delta_w(a_w) \right). \tag{1}$$

Finally, we use a_i^* for the equilibrium advertising volume of business i. We focus on the following two kinds of distortions:

Definition 1. An equilibrium entails *double marginalization* if the total disutility at the equilibrium is strictly greater than the total disutility at the industry optimum.

Definition 2. An equilibrium entails *misplacement* if the platform and the websites can jointly deviate and strictly increase industry profits while keeping total disutility at the equilibrium level.

Proposition 1. *Any equilibrium entails double marginalization, and the total disutility strictly increases in the number n of websites.*

Exposing visitors to more ads may increase the advertising revenue of a website. However, placing more ads deters some consumers from visiting the platform and potentially visiting other websites, In equilibrium, websites fail to internalize this negative externality, leading to the standard double marginalization. The severity of double marginalization—quantified by the total disutility $\delta_p + \frac{1}{n} \sum_w \delta_w$—increases in the number of websites. This observation resembles a Cournot oligopoly, in which the total output increases with an increase in the number of competing firms, even though each firm's individual output decreases in equilibrium.

The following result states that the equilibrium generically entails misplacement:

Proposition 2. *Unless the parameters satisfy $\gamma_p = n^{k-1} \gamma_w$ for all $w \in W$, any equilibrium entails misplacement. Moreover, if $\gamma_w = \gamma$ for all $w \in W$ and $\frac{1}{n}\gamma < \gamma_p < \gamma$, then the severity of misplacement increases in n, in that for every $w \in W$, $a_w^* - a_p^*$ is positive and increasing in n in equilibrium, whereas $a_w^{\Pi} - a_p^{\Pi}$ is negative and constant in n at the industry optimum.*

To see the intuition, suppose that websites 1 and 2 impose disutilities of 1 and 2 per unit of ads, respectively.[7] For example, website 2 may embed ads in a video, which consumers find more distracting. In equilibrium, website 2 chooses a positive advertising volume to ensure a positive advertising revenue. However, the businesses could increase their joint profit without changing the total disutility that consumers incur if website 1 alone placed ads (i.e., $a_1 = a_1^* + a_2^*$), because doing so minimizes consumer disutility and maximizes their visits, given the total disutility.

The second part of the result provides a condition under which the gap between the equilibrium and the industry optimum—in terms of the allocation of advertising volume—becomes wider as the number of websites increases. Specifically, under the stated condition, the platform should place more ads at the industry optimum, but in equilibrium, the websites place more ads, and as n increases, they place even more ads while the platform places fewer ads. Specifically, under the stated condition, the platform should place more ads than each website at the industry optimum. However, in equilibrium, the websites place more ads than the platform; moreover, as n increases, the websites place even more ads while the platform places fewer ads.

We define double marginalization and misplacement in terms of industry profit. However, the results have implications for efficiency because any outcome with either of these properties is Pareto dominated. For example, in equilibrium with misplacement, websites can increase the joint profit without changing total disutility, which also implies that they could strictly increase the joint profit and consumer surplus by adjusting $(a_i)_{i\in\overline{W}}$. Note that double marginalization or misplacement alone implies that the equilibrium is inefficient. Thus the equilibrium continues to be inefficient after changing the market structure or the websites' business models unless the change eliminates both distortions.

4 Competition and the Two Market Distortions

How to alleviate these distortions? We begin with a solution for double marginalization. Note that in our model, each website and the platform are complements; without the plat-

[7]For simplicity, this example uses linear disutility functions, which are outside the scope of our model.

form, consumers do not know the existence or relevance of each website; without websites, the platform—such as a search engine or a news aggregator—cannot direct consumers to relevant content. When a final good comprises complementary components, a well-known solution to the double marginalization problem is introducing competition in the markets for all but one of the individual components. For instance, suppose "hardware" and "software" are two perfectly complementary products produced by different firms. The firms face double marginalization, but if the hardware market became perfectly competitive, the software maker would be able to charge a price that implements the outcome that maximizes the industry profit.[8] Would the same logic apply to our setting?

To capture competition among websites, we assume that new websites enter the market, expanding the set of websites from W to $\hat{W}$. For each consumer, there are at least two relevant websites in $\hat{W}$ with probability 1. For example, for each website $w \in W$, there is another website $\hat{w} \in \hat{W} \setminus W$ that newly enters the market and has the same content, so that websites w and $\hat{w}$ are relevant to the same set of consumers. The platform now directs each consumer to a relevant website that sets the lowest advertising volume across all relevant websites. The standard logic of Bertrand competition ensures the existence of an equilibrium in which all websites choose $a_i = 0$. Taking the equilibrium choice of competing websites as given, the platform chooses a_p. In this setup, we obtain the following result:

Proposition 3. *(a) Competition eliminates double marginalization—i.e., the equilibrium total disutility under competition equals the one under the industry optimum. Thus competition increases consumer surplus.*

(b) Suppose that every website $w \in W$ has the same γ_w, denoted by γ. There is a $\gamma^* \geq 0$ that satisfies the following: Compared to the baseline model, competition increases the industry profit if and only if $\gamma \geq \gamma^*$. The profit comparison is strict whenever $\gamma \neq \gamma^*$.

Proposition 3-(a) confirms that competition eliminates double marginalization: Competi-

[8] Casadesus-Masanell, Nalebuff, and Yoffie (2007) and Cheng and Nahm (2007) study variations of such models for hardware and software. They study the case in which the producers in the sectors with competition are vertically differentiated. Some papers study the use of competition among firms in a particular category of a complementary bundle as a solution to the double marginalization problem. Dellarocas (2012) studies a related idea with performance-based fees in online advertising. More broadly, see, e.g., Rey and Tirole (1986), Shleifer and Vishny (1993), and Lerner and Tirole (2004, 2015).

tion forces websites to set $a_i = 0$ and enables the platform to act as a monopolist. Under Assumption 1, the problem of the platform choosing the total disutility coincides with the problem of maximizing industry profits.

At the same time, Proposition 3-(b) suggests that competition could exacerbate misplacement and reduce the industry profit. To see the intuition, consider a variant of our setup with linear disutility from advertisement. Suppose that the platform and each website impose disutilities of 1 and d per unit of ads, respectively. If $d > 1$, only the platform should place ads at the industry optimum. Such an outcome arises if websites face competition and are forced to set $a_i = 0$. However, if $d < 1$, competition between websites exacerbates misplacement: The platform should not place ads at the industry optimum when $d < 1$, but competition decreases the equilibrium advertising volume on websites and increases the ads on the platform. The resulting change still reduces total disutility and increases consumer visits; however, the advertising revenue may decrease because the increase in ads on the platform does not compensate for the decrease in ads on websites. Thus, competition between websites could reduce the industry profit when websites have access to effective advertising technology (i.e., a lower γ).

The negative impact of competition on the industry profit would not arise if consumers cared only about the total advertising volume, i.e., the total disutility is an increasing function of $a_p + \frac{1}{n}\sum_w a_w$. In such a case, under competition between websites, the platform can set $a_p = a_p^\Pi + \frac{1}{n}\sum_w a_w^\Pi$, which is the total advertising volume at the industry optimum without competition. By doing so, the platform can secure a profit that is weakly greater than the joint industry profit in the baseline model. Therefore our specification—that websites impose heterogeneous disutilities on visitors—is crucial for competition on one segment to decrease the industry profit. Point (b) of the proposition formalizes this intuition in terms of the disutility parameter, γ.

5 Micropayments

We return to the baseline model with the platform and n websites. Recall from Proposition 1 that an equilibrium generically entails misplacement, i.e., the websites and the platform can jointly adjust $(a_i)_{i\in\overline{W}}$ to increase the industry profit while keeping the total disutility at the equilibrium level. Can websites eliminate misplacement without such explicit coordination? One way is that websites use another strategic variable to equalize their equilibrium marginal disutilities. In this spirit, we extend the model by allowing websites to not only place ads but also to charge or subsidize consumers using per-visit *micropayments*. [9] To capture this idea, we focus on the case in which all websites can use micropayments. Appendix B studies an alternative case in which only some businesses can use micropayments.

Formally, we extend the baseline model as follows. Each business now chooses advertising volume $a_i \geq 0$ and price $t_i \in \mathbb{R}$. Prior to deciding whether to visit the platform, consumers observe (a_p, t_p) and $\frac{1}{n}\sum_{w\in W}[\delta_w(a_w)+t_w]$. If a consumer visits the platform and is directed to website w, the consumer receives an ex-post payoff of $v-\delta_p(a_p)-t_p-\delta_w(a_w)-t_w$. The payoffs to the platform and website w are $(a_p+t_p)m$ and $\frac{1}{n}(a_w+t_w)m$, respectively, where m is the mass of consumers who visit the platform. The way Nature selects relevant websites and the platform directs consumers to relevant websites remains the same.

We define double marginalization and misplacement as follows. First, we use $\Delta_i(a_i,t_i)$ (or Δ_i) for $\delta_i(a_i)+t_i$, which is the disutility—including the nuisance from ads and monetary transfer—that consumers incur from visiting business i. We then say that an equilibrium entails double marginalization if the equilibrium total disutility $\Delta_p+\frac{1}{n}\sum_{w\in W}\Delta_w$ is strictly greater than the one at the industry optimum.[10] An equilibrium entails misplacement if the websites can change $(a_i,t_i)_{i\in\overline{W}}$ to increase the industry profit while keeping total disutility at the equilibrium level.

[9] The micropayment scenario we consider differs from a subscription business model, in which users pay, for instance, a monthly fee in exchange for "all you can eat" access to a website. Instead, micropayments should affect each momentary decision of attention allocation, in a manner embodied, for instance, by the Basic Attention Token (BAT) offered by the Brave web browser.

[10] The total disutility at the industry optimum is the solution to the following problem: $\arg\max_{(a_i,t_i)_{i\in\overline{W}}}\left(a_p+t_p+\frac{1}{n}\sum_{w\in W}(a_w+t_w)\right)D\left(\delta_p(a_p)+t_p+\frac{1}{n}\sum_{w\in W}[\delta_w(a_w)+t_w]\right)$.

5.1 Micropayments can eliminate misplacement only.

The following result states that micropayments eliminate misplacement but not double marginalization.

Proposition 4. *In the game with micropayments, any equilibrium entails double marginalization but does not entail misplacement.*

The result of no misplacement under micropayment is intuitive. Suppose that in equilibrium, website w chooses a_w such that $\delta'_w(a_w) < 1$. Then it could profitably deviate by increasing advertising level a_w and decreasing t_w, while holding fixed total user disutility. Similarly, if $\delta'_w(a_w) > 1$, website w will have a profitable deviation.[11] A similar argument applies to the platform. As a result, the equilibrium with micropayment satisfies $\delta'_i(a_i) = 1$ for each business i. The marginal disutility of advertising equals the marginal disutility of money across all websites, and thus the equilibrium entails no misplacement. The result implies that misplacement crucially depends on the lack of another competitive instrument, a price for the service. To put it differently, the ad-financed revenue model drives misplacement. This result is related to the insight of Anderson and Coate (2005) in the sense that they pointed out that a lack of the Pigouvian corrective tax can generate some externality problems. In contrast, the micropayment does not eliminate double marginalization because websites do not internalize the effect of increasing a_i or t_i on the others' profits.

5.2 Welfare effects of micropayments are ambiguous.

The following result and subsequent discussion show that under Assumption 1 the introduction of micropayments can increase or decrease consumer surplus and industry profits.

Proposition 5. *Under Assumption 1, consumer surplus is greater in the game with micropayments than in the baseline model if and only if* $\left(\frac{1}{\gamma_p}\right)^{\frac{1}{k-1}} + \frac{1}{n}\sum_{w\in W}\left(\frac{1}{\gamma_w}\right)^{\frac{1}{k-1}}$ *is above some threshold. Industry profit is greater in the game with micropayments if* $\min_{i\in\overline{W}}\gamma_i$ *is sufficiently large.*

[11] A similar finding appears in the literature studying free-to-air versus subscription television (Choi, 2006; Crampes, Haritchabalet, and Jullien, 2009; Peitz and Valletti, 2008).

The result implies that when the advertising technology is ineffective (i.e., each γ_i is large), the introduction of micropayments reduces consumer surplus and increases industry profits because micropayments enable businesses to reduce the advertising volume and charge consumers for access to extract more surplus from consumers.

While Proposition 5 also highlights the potential benefit of micropayments for the industry, micropayments do not necessarily increase total profits when the stated condition is violated. Appendix B provides a numerical example in which the introduction of micropayments decreases industry profits and consumer surplus. In the example, the businesses are symmetric so that the equilibrium without micropayments does not entail misplacement. The introduction of micropayments reduces the equilibrium advertising volume, but each website also charges a positive price to consumers, which leads to a higher total disutility than without micropayments. The example highlights that micropayments exacerbate double marginalization and hurt both consumers and websites.

A natural question is how Propositions 4 and 5 would change when some businesses adopt micropayments and other businesses do not. The result that micropayments eliminate misplacement depends crucially on all websites the platform adopting them. Equilibrium can still entail misplacement when only one website adopts micropayments.[12] Indeed, Appendix B shows that the adoption of micropayments by only one website could exacerbate misplacement—e.g., it is possible that the game without micropayments has no misplacement, but the game in which only one business can use micropayments typically entails misplacement. The same appendix also shows that if the website that can use micropayments has an efficient advertising technology (i.e., a small γ_i), consumer surplus and the payoff to the other website increase, compared to the case with no micropayments.

6 Discussion

We discuss the implication of our results on digital markets in practice. First, our results highlight the trade-off between mitigating double marginalisation and misplacement that policymakers or businesses face. In a standard vertical chain of monopolies, introducing intense

[12] See Example 2 in Appendix B.

competition at one level of the chain, either upstream or downstream, restores the industry optimum and improves efficiency. However, in our model, although intense competition among websites eliminates double marginalization, it can also worsen the harms from misplacement. On the other hand, introducing micropayments, charged per visit by websites to consumers, can correct misplacement, but may worsen double marginalization. Moreover, the effects of adopting micropayments on consumer surplus and industry profit depend on parameters.

The trade-offs identified in our model have policy implications for any online markets involving an ads-revenue business model. In the traditional markets of complementary products like 'nuts and bolts', either a vertical integration into one entity or introducing competition into one segment of the two products can equally address the double marginalization problem. However, this conventional wisdom can be misleading in online advertising markets due to the misplacement effect. Vertical integration can replicate the joint profit maximization under which both double marginalization and misplacement can be resolved. By contrast, even the strongest competition à la Bertrand among the websites may makemisplacement worse. This is particularly so when the marginal disutility from a small website's advertising is lower than that from the dominant portal site. Since a higher marginal revenue from placing ads leads to a lower marginal disutility, the potential conflicts between double marginalization and misplacement are less likely to arise when the portal site's marginal ads revenue is high enough. In the opposite case, policy-makers need more caution.

From the alternative interpretation of the collection of personal data (cf. Section 2.2), we could interpret the ads as a level of data collection and the disutility as a privacy concern. The trade-offs we highlighted could be relevant to this context especially when the dominant platform's data policy generates users' greater privacy concerns. In a sense, our model reminds us of burgeoning information externality literature (e.g., Choi, Jeon, and Kim (2019); Acemoglu, Makhdoumi, Malekian, and Ozdaglar (2022)) because the websites end up collecting too much personal information when they fail to coordinate their actions. As our model suggests, bringing intense competition into the websites would not cure the misplacement of data collection; even if the platform, whose privacy harm is greater, may end up collecting too much personal information and thus incurs excessive privacy loss than intended.

Finally, a certain policy or change in market structure could resolve both double marginalization and misplacement. For example, if the platform and n websites merge to become a single entity, the resulting business chooses advertising volumes so that neither double marginalization nor misplacement will occur. However, it would likely be misleading to use this observation alone as a basis for supporting such a merger, as our model, deliberately simplified to focus on the two specific distortions, does not capture potential harms–such as foreclosure—that such a merger could entail.

7 Conclusion

Internet users frequently visit multiple ad-funded websites that play a complementary role in fulfilling their needs. This paper shows that, in such settings, websites have a tendency to distort their advertising policies in two ways. The first distortion is classic double marginalization, reflected by an excessive total amount of advertising. The second, novel distortion is what we call *misplacement*. When there is misplacement, the websites could have jointly reallocated advertisement among themselves and increased their total profit, without changing either their output levels (i.e., the number of users they each serve) or users' payoffs. We have shown that a particular, standard policy in vertical markets of increasing competition at one level of the chain can worsen the misplacement distortion even as it mitigates double marginalization in the usual way. We have also shown that if websites are able to charge per-visit micropayments to users, this eliminates misplacement but not double marginalization. Therefore, both of these two relatively straightforward interventions come with their own potential tradeoffs.

We believe that, broadly speaking, understanding the costs and benefits of (de)-centralization in online advertising is important for platforms and society. This is especially true in the current context in which jurisdictions such as Australia, Canada and the U.S. have enacted or proposed so-called 'link taxes' that would apply in circumstances like those contemplated by our model.[13] Under such policies, large platforms can be required to make a payment to a publisher each time they link to its site. In response to such proposals, platforms such as Google and Facebook have threatened to substantially curtail their activities in these jurisdictions. To

[13] See https://arstechnica.com/tech-policy/2023/06/google-tells-canada-it-wont-pay-link-tax-will-pull-news-links-from-search/

the best of our knowledge, such policies do not allow the platforms to condition such payments on publishers' ad levels, and thus, they do not appear to directly address the distortions we identify. As such, we hope this paper might help contribute towards policymakers' understanding of the relevant economic forces that underlie these highly significant online environments.

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Appendices

A Omitted Proofs

Proof of Proposition 1. First, we derive the total disutility at the industry optimum. The industry optimum is the solution to the following problem:

$$\max \left(a_p + \frac{1}{n} \sum_{w \in W} a_w \right) D \left(\delta_p(a_p) + \frac{1}{n} \sum_{w \in W} \delta_w(a_w) \right) \tag{A.1}$$

For each $i \in \overline{W}$, the first-order condition with respect to a_i is

$$D + \left(a_p + \frac{1}{n} \sum_{w \in W} a_w \right) D' \delta_i'(a_i) = 0.$$

Assumption 1 implies $a\delta_i' = k\delta_i$.

$$a_i D + \left(a_p + \frac{1}{n} \sum_{w \in W} a_w \right) D' k \delta_i(a_i) = 0.$$

Because this equation holds for every $i \in \overline{W}$, we obtain

$$\left(a_p + \frac{1}{n} \sum_{w \in W} a_w \right) D + \left(a_p + \frac{1}{n} \sum_{w \in W} a_w \right) \left(\delta_p(a_p) + \frac{1}{n} \sum_{w \in W} \delta_w(a_w) \right) D' k = 0.$$

Note that $\frac{D}{D'} = -\frac{1-F}{f}$. Setting $g = \frac{1-F}{f}$, we obtain

$$\delta_p(a_p) + \frac{1}{n} \sum_{w \in W} \delta_w(a_w) = \frac{1}{k} g \left(\delta_p(a_p) + \frac{1}{n} \sum_{w \in W} \delta_w(a_w) \right) \tag{A.2}$$

Therefore, the total disutility at the industry optimum is Δ^{Π} that solves

$$\Delta^{\Pi} = \frac{1}{k} g(\Delta^{\Pi}). \tag{A.3}$$

In equilibrium, each business $i \in \overline{W}$ solves

$$\max_{a_i} a_i D\left(\delta_p(a_p) + \frac{1}{n}\sum_{w\in W}\delta_w(a_w)\right), \tag{A.4}$$

taking the equilibrium choices of other businesses, $(a_j)_{j\neq i} = (a_j^*)_{j\neq i}$, as given. The first-order condition and $a\delta_i'(a) = k\delta_i(a)$ implies

$$\delta_p(a_p^*) = \frac{1}{k} g\left(\delta_p(a_p^*) + \frac{1}{n}\sum_{w}\delta_w(a_w^*)\right) \tag{A.5}$$

and

$$\frac{1}{n}\delta_w(a_w^*) = \frac{1}{k} g\left(\delta_p(a_p^*) + \frac{1}{n}\sum_{w}\delta_w(a_w^*)\right). \tag{A.6}$$

for the platform and each website w, respectively. Therefore, it follows

$$\delta_p(a_p^*) + \frac{1}{n}\sum_{w}\delta_w(a_w^*) = \frac{n+1}{k} g\left(\delta_p(a_p^*) + \frac{1}{n}\sum_{w}\delta_w(a_w^*)\right). \tag{A.7}$$

Thus the equilibrium total disutility Δ^* satisfies

$$\Delta^* = \frac{n+1}{k} g(\Delta^*). \tag{A.8}$$

Comparing equations (A.3) and (A.8) and noting that g is decreasing, we conclude that the total disutility at equilibrium is greater than at the industry optimum, and the former is increasing in n. $\square$

Proof of Proposition 2. If a profile $(a_i^*)_{i\in\overline{W}}$ of advertising volume entails no misplacement, then the businesses cannot increase their joint profit by reallocating ad placements while maintaining the same level of total disutility, $\Delta = \delta_p(a_p^*) + \frac{1}{n}\sum_{w\in W}\delta_w(a_w^*)$. Thus $(a_i^*)_{i\in\overline{W}}$ is a solution to the following problem:

$$\begin{aligned} &\max a_p + \frac{1}{n}\sum_{w\in W} a_w \\ \text{subject to} \quad &\delta_p(a_p) + \frac{1}{n}\sum_{w\in W}\delta_w(a_w) = \Delta. \end{aligned}$$

The Lagrangian method implies that $(a_i^*)_{i\in\overline{W}}$ entails no misplacement if and only if

$$\delta_p'(a_p^*) = \delta_w'(a_w^*), \forall w \in W.$$

We now examine when equilibrium entails misplacement. By Assumption 1, we can rewrite equation (A) as $\frac{\delta_p(a_p^*)}{a_p^*} = \frac{\delta_w(a_w^*)}{a_w^*}$ for all $w \in W$. Equations (A.5) and (A.6) then imply $a_w^* = na_p^*$. This is the condition under which the equilibrium does not entail misplacement. Setting $c := \frac{1}{k}g\left(\delta_p(a_p^*) + \frac{1}{n}\sum_w \delta_w(a_w^*)\right)$ and using equations (A.5) and (A.6), we have the equilibrium advertising level of each business:

$$a_p^* = \left(\frac{X}{\gamma_p}\right)^{\frac{1}{k}} \quad \text{and} \quad a_w^* = \left(\frac{nX}{\gamma_w}\right)^{\frac{1}{k}}$$

These equilibrium choices satisfy the condition for misplacement iff $\left(\frac{nX}{\gamma_w}\right)^{\frac{1}{k}} = n\left(\frac{X}{\gamma_p}\right)^{\frac{1}{k}}$ which reduces to $\gamma_p = n^{k-1}\gamma_w$. Therefore unless $\gamma_p = n^{k-1}\gamma_w$, the equilibrium entails misplacement.

Finally, we show the second part of the result by setting $\gamma_w = \gamma$ for all $w \in W$. We have $a_p^* < a_w^*$ in equilibrium whenever $\frac{1}{n}\gamma < \gamma_p$. So long as this inequality holds for some n, it continues to hold for all $n' \geq n$. As the number n of websites increases, the equilibrium total disutility increases. Equation (A.6) then implies that $\delta_p(a_p^*)$ decreases, which implies that the equilibrium disutility chosen by each website increases. In contrast, the industry optimum requires that a_p^* and $a_w^* = a^*$ stay constant for all n. The condition for no misplacement (i.e., $\delta_p'(a_p^*) = \delta_w'(a^*)$) then implies $a_p^* > a_w^*$ holds if $\gamma_p < \gamma_w$. To sum up, if $\frac{1}{n}\gamma < \gamma_p < \gamma$, the industry optimum requires that the platform places more ads, but in equilibrium, each website places more ads than the platform, and this gap increases in the number n of websites.

$\square$

Proof of Proposition 3. We first show Point (a). As we explain in the main text, when websites compete with each other and set zero advertising volume, the platform chooses a_p to maximize

$$\max_{a_p} a_p D(\delta_p(a_p)). \tag{A.9}$$

The equilibrium choice a_p^C of the platform satisfies

$$a_p^C = \frac{g(\delta_p(a_p^C))}{\delta_p'(a_p^C)} \Rightarrow \delta_p(a_p^C) = \frac{1}{k} g(\delta_p(a_p^C)). \tag{A.10}$$

Comparing this condition with that for the industry optimum (A.3), we conclude that the equilibrium total disutility coincides with the industry optimum. Thus competition decreases the equilibrium total disutility and increases consumer surplus.

To show Point (b), we show that the industry profit in the baseline model (i.e., no competition) decreases in γ. Equation (A.8) implies that the total equilibrium disutility $\Delta^i *$ is independent of (γ_p, γ). Equation (A.6) then implies that the equilibrium disutility coming from each website is independent of (γ_p, γ). As a result, a higher γ means that websites place fewer ads in equilibrium, leading to a lower industry profit. As $\gamma \to 0$, the industry profit diverges to ∞. As $\gamma \to \infty$, the industry profit at equilibrium is $a_p^*[1 - F(\Delta^*)]$ without competition and $a_p^C[1 - F(\delta_p(a_p^C))]$ under competition. Comparing (A.8) and (A.10), we obtain $\Delta^* > \delta_p(a_p^C)$. Under competition, the platform can choose a that satisfies $\delta_p(a) = \Delta^*$ and secure a payoff of $a[1 - F(\Delta^*)] > a_p^*[1 - F(\Delta^*)]$. Thus we have $a_p^*[1 - F(\Delta^*)] < a_p^C[1 - F(\delta_p(a_p^C))]$. Therefore, there is a unique positive threshold γ^* such that the industry profit is greater under competition if and only if $\gamma \geq \gamma^*$. $\square$

Proof of Proposition 4. Take any website $w \in W$, and let Δ_{-w}^* denote the total disutility imposed by all businesses but w. In equilibrium, website w solves the following problem:

$$\max_{a_w,t_w}(a_w+t_w)D\left(\Delta^*_{-w}+\frac{1}{n}(t_w+\delta_w(a_w))\right).$$

The first-order conditions with respect to a_w and t_w are

$$D\left(\Delta^*_{-w}+\frac{1}{n}(t_w+\delta_w(a_w))\right)+(a_w+t_w)\frac{1}{n}\delta'_w(a_w)D'\left(\Delta^*_{-w}+\frac{1}{n}(t_w+\delta_w(a_w))\right)=0 \quad \text{and}$$
$$D\left(\Delta^*_{-w}+\frac{1}{n}(t_w+\delta_w(a_w))\right)+(a_w+t_w)\frac{1}{n}D'\left(\Delta^*_{-w}+\frac{1}{n}(t_w+\delta_w(a_w))\right)=0,$$

respectively. These equations imply $\delta'_w(a^*_w)=1$. Note that a^*_w does not depend on t_w. As a result, if website w imposes disutility $\Delta_w=\delta_w(a_w)+t_w$, then payment t_w is determined by $t_w=\Delta_w-\delta_w(a^*_w)$. We can now write website w's problem in equilibrium in terms of the choice of Δ_w:

$$\max_{\Delta_w}(\Delta_w+a^*_w-\delta_w(a^*_w))D\left(\Delta^*_{-w}+\frac{1}{n}\Delta_w)\right).$$

The first-order condition is

$$D\left(\Delta^*_{-w}+\frac{1}{n}\Delta_w\right)+(\Delta_w+a^*_w-\delta_w(a^*_w))\frac{1}{n}D'\left(\Delta^*_{-w}+\frac{1}{n}\Delta_w\right)=0,$$

which implies

$$g(\Delta^*)=\frac{1}{n}\left(\Delta^*_w+a^*_w-\delta_w(a^*_w)\right). \tag{A.11}$$

This equation determines the equilibrium disutility imposed by website w as a function of the equilibrium total disutility. For the platform, the corresponding equation is

$$g(\Delta^*)=\Delta^*_p+a^*_p-\delta_p(a^*_p). \tag{A.12}$$

Adding up equation (A.11) for each $w\in W$ and equation (A.12) and using the definition

$\Delta^* = \Delta_p^* + \frac{1}{n}\sum_{w\in W}\Delta_w^*$, we obtain the equation that determines equilibrium total disutility:

$$(1+n)g(\Delta^*) = \Delta^* + a_p^* - \delta_p(a_p^*) + \frac{1}{n}\sum_{w\in W}[a_w^* - \delta_w(a_w^*)]. \tag{A.13}$$

The problem for the industry profit maximization is

$$\max_{\Delta}\left(\Delta + a_p^* - \delta_p(a_p^*) + \frac{1}{n}\sum_{w=1}^{n}[a_w^* - \delta_w(a_w^*)]\right)D(\Delta).$$

Here, a_i^* is the unique solution to $\delta_i'(a_i^*) = 1$; we use the observation that $\delta_i'(a_i) = 1$ must hold at the industry optimum as well. The first-order condition with respect to Δ is

$$g(\Delta^\Pi) = \Delta^\Pi + a_s^* - \delta_w(a_s^*) + \frac{1}{n}\sum_{w=1}^{n}[a_w^* - \delta_w(a_w^*)]. \tag{A.14}$$

Comparing equation (A.13) with (A.14) and noting that g is decreasing, we conclude $\Delta^* > \Delta^\Pi$, i.e., the equilibrium entails double marginalization.

To show that no equilibrium entails misplacement, suppose that the players jointly maximize the industry profit subject to the constraint that the total disutility is Δ, i.e., the businesses solve

$$\max a_p + t_p + \frac{1}{n}\sum_{w\in W}(a_w + t_w)$$
$$s.t. \quad \delta_s(a_s) + t_s + \frac{1}{n}\sum_{w\in W}(\delta_w(a_w) + t_w) = \Delta.$$

Solving this problem, we obtain $\delta'(a_i) = 1$ for all $i \in \overline{W}$. Because the equilibrium choice satisfies the above condition, the players cannot increase total profits while keeping the total disutility constant. Thus, no equilibrium entails misplacement. $\square$

Proof of Proposition 5. The proof of Proposition 3 shows that in the baseline model without micropayment, the equilibrium total disutility and consumer surplus are independent of

$(\gamma_i)_{i\in\overline{W}}$. With micropayment, the equilibrium choice of a_i^* satisfies $\delta_i'(a_i^*) = 1$, or equivalently, $a_i^* = \left(\frac{1}{k\gamma_i}\right)^{\frac{1}{k-1}}$. Also, $\delta_i'(a_i^*) = 1$ and $\delta_i(a_i) = \frac{1}{k}a_i\delta'(a_i)$ imply $\delta_i(a_i^*) = \frac{1}{k}a_i^*$, leading to $a^* - \delta_i(a_i^*) = \left(1 - \frac{1}{k}\right)a_i^*$. Equation (A.13) becomes

$$(1+n)g(\Delta^*) = \Delta^* + \left(1 - \frac{1}{k}\right)\left(a_p^* + \frac{1}{n}\sum_{w\in W} a_w^*\right)$$
$$= \Delta^* + \left(1 - \frac{1}{k}\right)\left(\frac{1}{k}\right)^{\frac{1}{k-1}}\left(\left(\frac{1}{\gamma_p}\right)^{\frac{1}{k-1}} + \frac{1}{n}\sum_{w\in W}\left(\frac{1}{\gamma_w}\right)^{\frac{1}{k-1}}\right) \tag{A.15}$$

This equation implies that for a fixed k, the equilibrium total disutility with micropayment is decreasing in $\left(\frac{1}{\gamma_p}\right)^{\frac{1}{k-1}} + \frac{1}{n}\sum_{w\in W}\left(\frac{1}{\gamma_w}\right)^{\frac{1}{k-1}}$, which completes the proof of the first part.

To show the second part, let q satisfy $(1+n)g(q) = q$. Equation (A.15) implies that as $(\min_{i\in\overline{W}}\gamma_i) \to \infty$, we have $\Delta^* \to q$ and $a_i^* \to 0$ for every i, so the industry profit converges to $qD(q) > 0$. Without micropayment, the industry profit converges to 0 as $(\min_{i\in\overline{W}}\gamma_i) \to \infty$, because the total disutility is constant while the advertising volume of each business approaches 0. Therefore for a sufficiently large $(\min_{i\in\overline{W}}\gamma_i) \to \infty$, industry profit is greater in the game with micropayments. □

B Appendix for Section 5: The Impact of Micropayments

Micropayments Can Decrease Industry Profits

Throughout this exercise, we assume $n = 1$. As a result, the game consists of consumers, the platform, and one website. Under Assumption 1, the primitives of the model are $(k, \gamma_p, \gamma_1, F)$. The following example shows that there is some $(k, \gamma_1, \gamma_2, F)$ such that the industry profit and consumer surplus decrease when the businesses can use micropayments.

Example 1. Suppose $k = 3$, $\gamma_p = \gamma_1 = 0.01$, and v is uniformly distributed between 0 and $\overline{v} = 10$. With micropayments, equilibrium total disutility Δ^P satisfies equation (A.15), which becomes

$$\Delta^P + \frac{40}{3}\cdot\frac{1}{\sqrt{3}} = 2(10 - \Delta^P),$$

which implies $\Delta^P = \frac{20(1-2\cdot\frac{1}{3\sqrt{3}})}{3} \approx 4.1$. The industry profit is then $2(10-\Delta^P)(1-\frac{\Delta^P}{10}) = 2\cdot\frac{10+40\cdot\frac{1}{3\sqrt{3}}}{3}\cdot\frac{1+4\cdot\frac{1}{3\sqrt{3}}}{3}$. Without micropayments, the total disutility Δ^N solves $\Delta^N = \frac{2}{k}g(\Delta^N)$, which now becomes $\Delta^N = \frac{2}{3}(10-\Delta^N)$. Thus we obtain $\Delta^N = 4 < \Delta^P$. Because each business chooses a such that $\delta_i(a) = 2$, we have $a_i = 200^{1/3}$. Thus the industry profit is $2\cdot 200^{1/3}\cdot(1-\frac{4}{10})$. Given the values for the industry profits and the total disutility with and without micropayments, we can numerically verify that the introduction of micropayments decreases the industry profit and consumer surplus.

Adoption of Micropayments by One Business

The following result and example study the setting in which only the platform can use micropayments. Below, we refer to this as the *partial micropayment* setting.

Proposition 6. *Suppose Assumption 1 holds. Compare the equilibrium of (i) the baseline with that of (ii) the partial micropayment setting. Consumer surplus under (i) is greater than that under (ii) if and only if γ_p is below some threshold. Also, the website obtains a higher payoff under (i) than (ii) if and only if γ_p is below some (possibly different) threshold. Moreover, if γ_p is sufficiently small, the industry profit is greater under (i) than (ii).*

Proof. Suppose only the platform uses micropayments, and let Δ denote the equilibrium total disutility. In equilibrium, a_1^* satisfies $a_1^* = \frac{g(\Delta)}{\delta_1'(a_1^*)}$, which reduces to

$$\delta_1(a_1^*) = \frac{g(\Delta)}{k}. \tag{A.16}$$

The first-order condition of the platform is

$$\hat{\delta}_p + \left(1-\frac{1}{k}\right)\left(\frac{1}{k\gamma_p}\right)^{\frac{1}{k-1}} = g(\Delta).$$

Summing up the two equations, we obtain

$$\Delta + \left(1 - \frac{1}{k}\right)\left(\frac{1}{k\gamma_p}\right)^{\frac{1}{k-1}} = \left(1 + \frac{1}{k}\right) g\left(\Delta\right). \tag{A.17}$$

When no website uses micropayments, total disutility does not depend on (γ_p, γ_1) because of Assumption 1. After the platform adopts micropayments, total disutility is increasing in γ_p. Also, equation (A.16) implies that website 1 chooses a higher a_1^* when Δ decreases. Thus a lower γ_p increases consumer surplus and website 1's profit, which guarantees the existence of the thresholds. □

The above result provides a sufficient condition under which the partial adoption of micropayment increases industry profits. The following example shows that if this condition is violated, partial adoption may reduce industry profits.

Example 2. Suppose $\gamma_p = 0.01$, $\gamma_1 = 0.05$, $k = 2.9$, and v is uniformly distributed between 0 and $\overline{v} = 10$. Suppose only the platform can charge micropayments, and let Δ_U denote equilibrium total disutility. Equation (A.17) implies that

$$\Delta_U = \frac{\overline{v}(1+\frac{1}{k}) - (1-\frac{1}{k})(\frac{1}{k\gamma_1})^{\frac{1}{k-1}}}{2+\frac{1}{k}}$$

The payoff to the platform is

$$\Pi_p := g(\Delta_U)\left(1 - \frac{\Delta_U}{\overline{v}}\right) = (\overline{v} - \Delta_U)\left(1 - \frac{\Delta_U}{\overline{v}}\right).$$

The payoff to website 1 is

$$\Pi_1 := \left(\frac{g(\Delta_U)}{\gamma_1 k}\right)^{\frac{1}{k}}\left(1 - \frac{\Delta_U}{\overline{v}}\right) = \left(\frac{\overline{v} - \Delta_U}{\gamma_1 k}\right)^{\frac{1}{k}}\left(1 - \frac{\Delta_U}{\overline{v}}\right)$$

Under partial micropayments, the equilibrium profit is $\Pi_p + \Pi_1$, which we can numerically verify to be at least 5.8. In the total absence of micropayments, the analogous calculation as in

Example 1 yields that the industry profit in equilibrium is at most 5.79. Thus, in this example, partial micropayments strictly reduce industry profits. We can also numerically verify that partial micropayments benefit both the platform and consumers. Finally, in equilibrium under partial micropayments, we have $a_1^* \approx 3.6$ and $\delta_1'(a_1^*) \approx 1.7 > 1 = \delta_p'(a_p^*)$. Thus, misplacement persists in this setting.

A switch from the baseline model to a partial micropayments regime could exacerbate misplacement. To see this, suppose $\delta_p(\cdot) = \delta_1(\cdot) = \delta(\cdot)$. Without any micropayments, no equilibrium entails misplacement, because, given the symmetry of the game, the platform and website 1 choose the same ad volume. Under partial micropayments, as in (A.12), the first-order condition for website the platform is $\Delta_p^* + a_p^* - \delta(a_p^*) = g(\Delta_p^* + \delta(a_p^*))$. The first-order condition for website 1 is $a_1^* = \frac{g(\Delta_p^* + \delta(a_1^*))}{\delta'(a_1^*)}$. Suppose that the equilibrium does not entail misplacement, which implies $\delta'(a_1^*) = 1$ and thus $a_p^* = a_1^* = a^*$. Plugging these into website 1's first-order condition and combining this with the platform's we obtain $\Delta_p^* = \delta(a_p^*)$, i.e., the platform sets zero monetary transfer. Plugging back to the platform's first-order condition, we have $a^* = g(2\delta(a^*))$. Note that a^* is the solution of $\delta'(a^*) = 1$ and independent of F. Thus whenever distribution F fails to satisfy $a^* = g(2\delta(a^*))$, we obtain a contradiction, i.e., the equilibrium entails misplacement. For example, if F is the uniform distribution on $[0, \overline{v}]$, we have $g(x) = \overline{v} - x$. Except for the non-generic case of $\overline{v} = a^* + 2\delta(a_1^*)$, a switch to partial micropayments introduces misplacement.