Monopolistic Screening under Narrow Framing:
Applications to Product and Credit Markets

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Abstract

We extend the standard model of monopolistic screening to allow for some consumers who engage in narrow framing, a prominent behavioral bias. Narrow framing generates a bias toward high quality-price ratios, which induces even high-type consumers to choose the menu that targets low-type consumers. To strategically account for narrow framing, when the monopolist induces the high-type consumers to stay with the more expensive menu, there arises a downward quality distortion even at the top and a smaller downward distortion at the bottom. We also apply our analysis to optimal loan contracts to screen heterogeneous borrowers with different default risks.

Keywords: Narrow Framing; Non-linear Price Discrimination; Behavioral Contract Theory; Adverse Selection; Collateral

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1 Introduction

Contract theory is one of the major advances in information economics and mechanism design. Recently, studies of optimal contracting have begun to explore replacing the agent in classical principal-agent problems with an agent who is subject to systematic behavioral biases. Indeed, one of the most rapidly growing areas at the intersection of neoclassical and behavioral economics is on behavioral contract theory. In a survey article on this field, Köszegi (2014) highlights the importance of analyzing screening contracts with behavioral agents but with the purpose of studying the same questions that are of interest in the classical screening problems. Köszegi (2014, p. 1100) notes,

Screening seems to be an understudied topic in behavioral contract theory… More generally, there seems to be little research on how insights from psychology and economics affect classical screening problems in which the private information concerns a standard preference parameter.

In this paper we make one significant departure from the standard monopolistic screening model of price discrimination (e.g., Spence 1975; Mussa and Rosen 1978; Goldman et al. 1984; Maskin and Riley 1984) by considering heterogeneity in framing among buyers. Some consumers have ‘neutral framing,’ free from any behavioral biases; they are the standard consumers in the classical theory. By contrast, other consumers have ‘narrow framing’ under which they separately account for the utility gain of consumption from the utility loss of a payment, which is a form of mental accounting.

Narrow framing has been documented in experiments (Tversky and Kahneman 1981; Rabin and Weizsäcker 2009), and has been applied to explain the equity premium puzzle (Benartzi and Thaler 1995), and limited stock market participation (Barberis et al. 2006) in financial markets as well as sub-optimal behavior in insurance markets (Gottlieb and Mitchell 2015). Here, we apply this basic behavioral bias to consumer choice to study how it affects equilibrium behavior in the standard monopolistic screening problem and to see whether the standard conclusions are robust to this
behavioral bias.¹

Our analysis makes three main contributions: First, we show that the well-known conclusions of the standard screening model (e.g., no quality distortion at the top) no longer hold when some consumers in the market have this basic behavioral bias. Second, we show that narrow framing does not necessarily adversely affect social welfare. The overall effect of narrow framing on social welfare depends on the relative strength of two opposing forces: narrow framing reduces the firm’s profit by reducing the value of higher quality to narrow framing consumers relative to neutral framing, but the loss in profit to the firm is offset by an increase in consumer surplus. We find in numerical simulations that welfare can actually increase with narrow framing. These results challenge the intuition that (i) behavioral biases do not affect equilibrium prices, and (ii) that, to the extent behavioral biases do affect markets, they will reduce social welfare. Finally, we apply the model to analyze how narrow framing affects equilibrium behavior in credit markets and we find that low-risk buyers will face higher interest rates and lower collateral requirements relative to the standard case with only neutral framing agents.

While narrow framing has previously been applied in financial markets and insurance markets, in this paper we consider the implications of narrow framing for the optimal screening contract in product markets. A key component of the classical screening models for a seller-buyer relationship is how the seller optimally designs a contract so that a more profitable buyer does not choose a less profitable menu—captured by incentive constraints—but still elicits the consumers’ private information that enables the firm to identify the more profitable buyers. The optimization calls for a distortion in either the quality or quantity of traded goods, from the complete information first-best and for providing a strictly positive information rent to the more profitable buyer. In the classical optimal nonlinear price-quality problem the information rent, which the seller wants to minimize if possible, is an increasing function of the quality offered to the less profitable buyer.

¹For choices under risk, narrow framing pertains to evaluating risks in isolation. Our formulation of narrow framing in consumer choice involves evaluating product attributes in isolation. Schneider and Kim (2020) show that this formulation of narrow framing in consumer choice also predicts narrow framing in choices under risk as observed experimentally by Tversky and Kahneman (1981) and Rabin and Weizsäcker (2009).
As a result, the seller has an incentive to distort the quality to the less profitable buyer to minimize the information rent, which is often referred to as “downward distortion at the bottom,” whereas the first-best quality is offered to the more profitable buyer, a finding called “no distortion at the top.” In this paper we report that when a subset of the more profitable buyers, usually called 'high-type,' evaluate the utility with narrow framing, then there arise interesting twists to this standard reasoning.

We first show that a high-valuation consumer with narrow framing has an incentive to choose the menu designed for a low-valuation consumer (Proposition 1). This is because the low price/quality menu provides a larger quality-price ratio over the high price/quality menu targeting the high-valuation consumer when neutral framing (that is, standard) consumers are indifferent between the two menus. This implies that narrow framing plays the role of an additional incentive compatibility constraint to induce the more profitable consumer to purchase the original menu.

By attempting to address this change in preference due to narrow framing, the seller’s optimal strategy modifies the standard trade-off between rent extraction and allocative efficiency. We show that the information rent yielded to the high-valuation consumer depends not only on the quality for the low-valuation consumer, but now also on its own quality measure. We thus find that the quality for the high-type consumer can be distorted downward from its first-best level as well (Proposition 2). The intuition for this result is simple. As the narrow framing consumers’ preferences tilt towards a higher quality-price ratio, in order to keep the high-valuation narrow framing consumers with the “premium” menu, the monopolist needs to increase the premium menu’s quality/price ratio, which calls for a move toward reducing the quality and the price. We also show that a greater degree of narrow framing leads to a larger downward distortion at the top (Proposition 3).

Regarding the quality for the low-type consumer, we find that it still can be distorted downward from the first-best quality as usual, but the extent of this distortion decreases with a higher degree of narrow framing (Proposition 4 and 5). This result needs to be understood jointly with the downward distortion at the top. As the monopolist introduces a downward distortion even for the premium menu to accommodate the narrow framing consumers, the incentive to degrade the basic
menu to prevent the high-type from buying this basic menu has declined. As a result, the distortion arises in a smaller magnitude with narrow framing.

In addition to the usual price discrimination context, we illustrate how our analysis can be extended to study the role of narrow framing in a banking application where a monopoly lender screens borrowers with different credit risks. We find that the narrow framing of the high-risk type borrower generates externalities onto the low-risk type borrower by reducing the collateral requirement (Proposition 6). The intuition is as follows. The low-risk type borrower’s participation constraint is binding and her contract is determined by the lender’s incentive to prevent the high-risk type borrower from having an incentive to mimic the low-risk type. Since the high-risk type narrow framing borrower perceives that the cost associated with collateral is much higher due to her narrow framing, now the lender can screen out the high-risk type borrower with a smaller collateral (as a screening instrument) from the low-risk type. Because the collateral incurs social inefficiency due to its liquidation cost, the narrow framing improves social efficiency. Lastly, we also discuss several empirical predictions associated with the smaller collateral requirement under narrow framing in Section 6.5.

**Related Literature**  The paper most closely related to ours is the study of contracts with framing by Salant and Siegel (2018). They incorporate framing by supposing that sellers choose frames to temporarily increase consumer valuations and that consumers can renego on their purchases after the framing effect has worn off. Our approach is complementary to Salant and Siegel (2018): rather than letting the monopolist choose the frame, we study consumers who narrowly (or neutrally) frame their own choices, and the monopolist chooses its optimal strategy anticipating the effects of narrow framing. Other studies of monopolistic screening with behavioral consumers include Rezaei and De Jaegher (2015) and Hahn et al. (2018) who both study screening with loss-averse consumers. Both of these papers assume that consumers have expectation-based reference-dependent preferences of the kind proposed by Köszegi and Rabin (2007). This assumption leads to predictions that are the opposite of those implied by narrow framing. In particular, as Rezaei
and De Jaegher (2015) observe, if rich consumers expect to buy high quality and poor consumers expect to buy low quality, the Köszegi-Rabin model implies that high-type consumers will be less inclined to choose the low price-low quality menu than in the classical screening model.

Our approach which predicts high-type consumers to be more inclined to choose the low price-low quality menu than in the classical model is motivated by a robust narrow framing bias rather than a particular specification of the consumer’s reference point. The basic intuition that consumers are biased toward high quality-price ratios is also a general prediction of the salience-based model of consumer choice due to Bordalo et al. (2013) which arises from basic properties of perception. We thereby add to the literature on mechanism design with behavioral agents by grounding the model with a strong psychological foundation. We believe the setting we study is also important given the fundamental nature and economic relevance of a bias toward narrow framing.

While narrow framing has been primarily studied in choices under risk (lottery choices, financial markets, and insurance markets), it also influences mental accounting (Thaler 1985) and so is naturally relevant to other types of consumer decisions. In addition to the bias toward high quality-price ratios, Schneider and Kim (2020) show that narrow framing and diminishing sensitivity jointly produce other common biases in consumer choice, including an attraction to products bundled with free features (e.g., hotels with free breakfasts, entrees with free refills, and products with free shipping) and a bias toward buy-one-get-one free promotions. In the presence of narrow framing, the bias toward high quality/price ratios is implied by both diminishing sensitivity and by loss aversion of the prospect theory value function. However, loss aversion does not generate many of the other consumer biases that are implied by diminishing sensitivity (Schneider and Kim, 2020). Given the broader implications of diminishing sensitivity for biases in consumer choice and the limited role of loss aversion for routine consumer transactions (Thaler 1985; Novemsky and Kahneman, 2005), we focus on the role of diminishing sensitivity in our analysis.

The problem of nonlinear pricing that offers different menus of price-quality or price-quantity
pairs to consumers has been a classic problem in microeconomics. Since our results show that the standard result of ‘no distortion at the top’ and ‘downward distortion at the bottom’ is challenged in the presence of narrow framing, this paper is complementary to several works that identify similar challenges in a variety of model setups. Specifically, departures from the standard distortion pattern have often been reported in the literature in one-sided markets with network effects (e.g., Csorba 2008; Hahn 2003; Meng and Tian 2008; Sundararajan 2004), the literature on countervailing incentives (e.g., Lewis and Sappington 1989; Maggi and Rodriguez-Clare 1995) and the literature on two-sided price discrimination (e.g., Böhme 2016; Jeon et al. 2020).

2 A Screening Model with Narrow Framing

2.1 The Environment

We consider a standard monopolist screening model in which consumers prefer higher quality and lower prices but they may differ in their relative valuations for these two attributes. A consumer earns her indirect utility $u(\theta, q) - p$ if the consumer buys one unit of quality or quantity $q$ at price $p$ and zero if she does not buy, where $u(\theta, q)$ and $p$ are both measured in dollars. There are two types of buyers, ‘high type’ (H) and ‘low type’ (L) consumers, whose type reflects the consumer’s taste for quality $\theta \in \Theta$ where $\Theta = \{\overline{\theta}, \underline{\theta}\}$ is the set of types with $\overline{\theta} > \theta > 0$. The high-type consumers care more about quality than the low type, (i.e., $u(\overline{\theta}, q) > u(\theta, q)$ for any given $q$). We also assume that the high types derive a larger marginal utility from an incremental quality enhancement than the low types (i.e., $\partial u(\overline{\theta}, q)/\partial q > \partial u(\theta, q)/\partial q$), which is the standard single-crossing property. We denote by $\Delta \theta \equiv \overline{\theta} - \theta > 0$ the spread of uncertainty on the consumer’s type. All consumers commonly prefer higher quality, which means that $u(\theta, q)$ is an increasing function of $q$, (i.e., $\partial U/\partial q > 0$). We assume that a proportion, $\lambda \in (0, 1)$, of consumers are high types and $1 - \lambda$ are low types.

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The monopolist faces a convex production cost of $c(q)$ with $c' > 0$ and $c'' > 0$ for $q > 0$, and $c(0) = 0$ and $c'(0) = 0$. In words, it is more costly and increasingly more expensive to produce higher quality. The firm offers two menus $\{(\bar{p}, \bar{q}), (p, q)\}$ where the first menu $(\bar{p}, \bar{q})$ targets the high-type consumers with a higher price $\bar{p}$ and a superior quality $\bar{q}$, and the second menu $(p, q)$ is designed to attract the low type with a less expensive price with a lower quality. The timing of information and actions is as follows: consumers discover their type first, and then the monopolist offers two bundles of goods, $\{(\bar{p}, \bar{q}), (p, q)\}$, and lastly consumers make their purchase decisions. Like the manager-worker problem described by Laffont and Martimort (2002), under the first-best, the firm extracts all the consumer surplus because it has complete information. The results, $c'(q) = \partial u(\theta, q)/\partial q$ and $p = u(\theta, q)$ for $\theta \in \{\theta, \bar{\theta}\}$, fully characterize the firm’s decisions.

Now we consider one important deviation from the standard model by introducing heterogeneity in framing. Following Schneider and Kim (2020), we define a ‘frame’ as a way of formalizing how consumers aggregate their values for attributes of choice alternatives, which thereby allows for heterogeneity in mental accounting.

**Definition.** We refer to a frame that does not aggregate any attribute values as a *narrow frame* and a frame that aggregates all attribute values of an alternative as a *neutral frame*.

Let $\alpha$ represent the fraction of high-type consumers who engage in narrow framing. Let $v(\cdot)$ denote the prospect theory value function (Kahneman and Tversky 1979; Thaler 1985) for any attribute. Then, the neutral consumer will have the standard surplus of $v(u(\theta, q) - p)$, but the narrow framing consumer evaluates the product according to $v(u(\theta, q)) + v(-p)$. We assume that the prospect theory value function exhibits diminishing sensitivity with $v' > 0$ and $v'' < 0$ for any positive $p$. We also assume that $v(-p) = -v(p)$. The value function can also have a loss aversion parameter, but as observed by Thaler (1985) and Novemsky and Kahneman (2005), loss aversion is not present (or is much weaker) for money given up as intended in routine consumer purchases.\footnote{Note that we introduce narrow framing to high-type consumers, but all of low-type consumers are neutral type. This assumption simplifies the problem so much that we gain much tractability. However, the consideration of narrow framing to low-type consumers may change our results unless the reservation utility is set to zero.}
We thus rely only on diminishing sensitivity and narrow framing. Our results continue to hold under a form of loss aversion (defined as \(-v(-p) > v(p)\) for \(p > 0\)).

2.2 Frames and Choices

To illustrate one important consequence of framing in consumer choice, consider the decision of whether to choose Product H with the value of quality \(u(\theta, q)\) at a price \(p\), or Product L with the value of quality \(u(\theta, q)\) at a price \(p\). Suppose that \(u(\theta, q) - p = u(\theta, q) - p \geq 0\), which implies that under neutral framing all consumers are indifferent between H and L. However, we find that under narrow frames consumers are biased toward a product with the higher quality/price ratio.

To show this, let \(u(\theta, q) = u(\theta, q) + k_1\) and \(p = p + k_2\) where \(k_1, k_2 > 0\). Suppose that Product H has a higher quality/price ratio than Product L. The indifference between the two products, \(u(\theta, q) - p = u(\theta, q) - p\), leads to \(k_1 = k_2 = k\). Then we would have

\[
\frac{u(\theta, q)}{p} = \frac{u(\theta, q) + k_1}{p + k_2} = \frac{u(\theta, q) + k_{(k_1=k_2)}}{p + k} > \frac{u(\theta, q)}{p}.
\]

The inequality of (1) is simplified as \(p > u(\theta, q)\), which contradicts \(u(\theta, q) - p \geq 0\). This means that Product L must have the higher quality-price ratio compared to Product H when the two products yield the same surplus under the neutral frame.

**Lemma 1.** The lower price-quality offer has a higher quality-price ratio than the higher price-quality alternative when a neutral framing consumer is indifferent between the two offers in the menu.

How would the narrow framing agent find the comparison? A consumer who engages in narrow framing will choose L over H, given that a neutral framing consumer with the same taste for quality is indifferent between L and H. A mathematical proof for this finding is brief. A preference for L over H in narrow frames implies

\[
v(u(\theta, q)) - v(p) < v(u(\theta, q)) - v(p),
\]
which yields
\[ v(u(\theta, \bar{q})) + v(\bar{p}) > v(u(\theta, \bar{q})) + v(p). \]

This inequality holds by concavity of \( v \) in the gain domain, since \( u(\theta, \bar{q}) > \bar{p} > p \) and \( u(\theta, \bar{q}) > \bar{p} \), and \( u(\theta, q) + \bar{p} = u(\theta, \bar{q}) + p \). The result straightforwardly generalizes to the case of loss aversion with \( v(x) < -v(-x) \). Combined with Lemma 1, we have the following.

**Proposition 1** (Narrow framing on menu choices). Suppose that the neutral framing consumers find the two menus \((q, p)\) and \((\bar{q}, \bar{p})\) indifferent. Then, the consumers with narrow framing will strictly prefer the low quality and low price menu \((q, p)\) which gives the higher quality-price ratio than the high quality and high price menu \((\bar{q}, \bar{p})\) which gives the lower quality-price ratio.\(^4\)

This result provides a crucial starting point for the optimal screening with the narrow framing consumers. When the incentive compatibility (IC) constraint is binding for a high-type consumer who employs neutral frames in the sense that \( u(\theta, q) - p = u(\theta, \bar{q}) - \bar{p} \), a narrow framing high-type consumer finds it better to choose \((p, q)\) over \((\bar{p}, \bar{q})\). This implies that the firm needs to provide more favorable terms to induce the high-type narrow framing consumer into choosing the originally targeted H menu \((p, q)\) by offering at least \( v(u(\theta, \bar{q})) - v(p) = v(u(\theta, q)) - v(p) \). The firm needs to find an optimal contract facing the trade-off associated with inducing the narrow framing consumer into the H menu or alternatively leaving the narrow framing consumer to choose the L menu. What would be the firm’s optimal strategy? How would the firm’s strategy affect the decisions of the standard neutral framing high-type consumers? What are the welfare effects of narrow framing on firm profits and consumer surplus? We study these questions in the following sections.

There is one important caveat before proceeding to the analysis. One might assume the principal offers three contracts, one for each type of consumer: low, neutral high, and narrow high. However, we assume that the seller offers two contracts, one for low types and one for high types. This is intentional since we are particularly interested in how narrow framing leads to deviations from the

\(^4\)We report a similar result in Schneider and Kim (2020). Here we apply it to a general vertical differentiation setting.
standard optimal menu with two contracts. That is, we focus on the trade-off arising from inducing the narrow framing consumers to choose the same menu as the ordinary high types. This enables us to directly relate our findings pertaining to quality distortions to the predictions of the standard model. It is also consistent with the recommendation in Köszegi (2014) to investigate how insights from psychology and economics affect the classical screening problem.

3 Benchmark Analysis: No Narrow Framing

We first review the second-best results for the standard setting with no narrow framing, and then examine the monopolist’s optimal contracting in the presence of heterogeneity in how consumers frame their purchases. Our presentation of this benchmark case is brief since this case is well known (e.g., Laffont and Martimort 2002; Bolton and Dewatripont 2005). Under the second-best, the monopolist does not know the consumer’s type. In order to prevent H type consumers from mimicking L type consumers, the monopolist faces the following incentive compatibility (IC) constraint:

\[ IC : u(\bar{\theta}, \bar{q}) - \bar{p} \geq u(\bar{\theta}, q) - p \]  

where the prospect theory value function \( v \) is not relevant for the neutral consumers and thus it is suppressed. Further, for both menus to be accepted, only the L type consumer’s individual rationality (IR) constraint is binding:

\[ IR : u(\underline{\theta}, \underline{q}) - p \geq 0 \]  

The monopolist solves the following problem:

\[
\max_{\{(p, q), (\bar{p}, \bar{q})\}} \pi = \lambda \left[ \bar{p} - c(\bar{q}) \right] + (1 - \lambda) \left[ p - c(q) \right]
\]
As both constraints (2) and (3) are binding, the problem can be written as the choice of $q$ and $\bar{q}$:

$$
\max_{q, \bar{q}} \pi = \lambda \left[ u(\bar{\theta}, \bar{q}) - (u(\bar{\theta}, q) - u(\bar{\theta}, q)) - c(\bar{q}) \right] + (1 - \lambda) \left[ u(\theta, q) - c(q) \right]
$$

(5)

The first order condition for profit maximization is for $\bar{q}$ given by

$$
\frac{\partial \pi}{\partial \bar{q}} = \lambda \left[ \frac{\partial u(\bar{\theta}, \bar{q})}{\partial \bar{q}} - c'(\bar{q}) \right] = 0,
$$

which yields the same characterization as the first-best: $c'(\bar{q}) = \frac{\partial u(\bar{\theta}, \bar{q})}{\partial \bar{q}}$ for the high-type.

Regarding $q$, we have

$$
\frac{\partial \pi}{\partial q} = (1 - \lambda) \left[ \frac{\partial u(\theta, q)}{\partial q} - c'(q) \right] - \lambda \left[ \frac{\partial u(\bar{\theta}, q)}{\partial q} - \frac{\partial u(\theta, q)}{\partial q} \right] = 0,
$$

which yields

$$
c'(q) = \frac{\partial u(\theta, q)}{\partial q} - \lambda \left[ \frac{\partial u(\bar{\theta}, q)}{\partial q} - \frac{\partial u(\theta, q)}{\partial q} \right] < \frac{\partial u(\theta, q)}{\partial q}
$$

where the inequality holds under the single crossing property. Thus, we confirm the standard result that $\bar{q}^{SB} = \bar{q}^{FB}$ and $q^{SB} < q^{FB}$. The intuition behind this result is that the monopolist has to give up some revenue by lowering the quality of the L menu to elicit the information about the consumers who are of type H.

The total social welfare generated under the second-best is given by:

$$
SW^{SB} = \pi^{SB} + CS^{SB} = \lambda \left[ u(\bar{\theta}, \bar{q}^{SB}) - c(\bar{q}^{SB}) \right] + (1 - \lambda) \left[ u(\theta, q^{SB}) - c(q^{SB}) \right]
$$

The L type consumers extract zero net surplus. However, the lower $\bar{p}$ leads to positive consumer surplus for H type consumers, which is caused by the positive information rent $u(\bar{\theta}, q) - u(\theta, q) > 0$. 

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4 Optimal Contracting with Narrow Framing

4.1 No Inducement Regime

The monopolist can always choose to not induce H type narrow framing consumers. Under a no inducement scheme, a narrow framer will buy the L menu as long as the high-type neutral consumer’s IC constraint is binding, i.e., \( u(\overline{\theta}, \overline{q}) - p = u(\overline{\theta}, q) - p \). Considering the same constraints under the standard second-best analysis where IC\(_H\) and IR\(_L\) are binding, the monopolist’s problem with no inducement can be written as:

\[
\max_{q, \overline{q}} \pi = \lambda (1 - \alpha) [u(\overline{\theta}, \overline{q}) - (u(\overline{\theta}, q) - u(\theta, q)) - c(\overline{q})] + (\lambda \alpha + (1 - \lambda)) [u(\theta, q) - c(q)]
\]

Now a fraction of \( \lambda (1 - \alpha) \) consumers who are the neutral H type consumers will buy the H menu, and the fraction of \( \lambda \alpha + (1 - \lambda) \), the sum of narrow framing H type consumers and all L type consumers, will buy the L menu. The analysis for the optimal qualities is exactly the same except that now different numbers of consumers buy the two menus. Again, the high-type consumers are offered the socially optimal (first-best) quality. The narrow framing high-type consumers and the low-type consumers are offered a quality that is distorted downward compared to the first best, but its degree of quality degradation is smaller relative to the case of no narrow framing:

\[
c'(q) = \frac{\partial u(\theta, q)}{\partial q} - \frac{\lambda (1 - \alpha)}{1 - \lambda + \alpha \lambda} \left[ \frac{\partial u(\overline{\theta}, q)}{\partial q} - \frac{\partial u(\theta, q)}{\partial q} \right]
\]

where the inequality holds for any \( \alpha \in (0, 1) \). The distortion at the bottom becomes smaller because intuitively now fewer H type consumers buy the H menu as the narrow framing H type consumers choose the L menu, which in turn means that it is less important to distort the quality for the L menu so as to make the H type consumer not to mimic the L type consumers.
4.2 Inducement Regime

Now consider the monopolist’s choice when he does induce the high-type narrow framing consumers to choose from the H menu. At the optimum, the H type neutral framing consumers will receive some “discounts” because of the modified IC constraint, while the IR constraint for the L type consumers will not be affected. Thus, the two relevant constraints are the following:

\[ IC_{\text{narrow}} : v(u(\theta, q)) - v(p) \geq v(u(\bar{\theta}, q)) - v(p) \]

\[ IR : u(\theta, q) - p \geq 0 \]

which modifies the monopolist’s problem as follows:

\[
\max_{\vec{q}, \vec{\theta}} \tilde{\pi} = \lambda \left[ v^{-1} \left( v(u(\theta, q)) - v(u(\bar{\theta}, q)) + v(u(\bar{\theta}, q)) \right) - c(q) \right] + (1 - \lambda) \left[ u(\theta, q) - c(q) \right]
\]

Note that the first order conditions with respect to \( \vec{q} \) call for the Kuhn-Tucker conditions:

\[
\frac{\partial \tilde{\pi}}{\partial q} \leq 0, \quad q \geq 0, \quad \text{and} \quad \frac{\partial \tilde{\pi}}{\partial \vec{q}} q = 0.
\]

Because of the standard monotonicity constraint (\( \vec{q} > \bar{q} \geq 0 \)), we must have \( \frac{\partial \pi}{\partial \vec{q}} = 0 \) to characterize \( \vec{q}^* \), using the asterisk to denote the inducement case. Next, to contrast the implications arising due to narrow framing with the standard case of no framing effect, consider the neutral framing benchmark. The high-type consumers with no narrow framing face the price of

\[ \bar{p} = u(\bar{\theta}, \bar{q}) - (u(\bar{\theta}, \bar{q}) - u(\theta, q)) \]

which means that the marginal effect of the H quality on the H price, \( \partial \bar{p}/\partial \bar{q} = \partial u(\bar{\theta}, \bar{q})/\partial \bar{q} \), is independent of the information rent. Since the information rent depends on the L quality \( q \) but not on the H quality \( \bar{q} \), the monopolist has no incentive to induce a quality distortion on \( \bar{q} \). Put
differently, the information rent yielded to the high-type consumers cannot be reduced via a quality adjustment to the H menu. However, this is no longer the case with narrow framing. To see this point, let us look at the derivative of $p$ with respect to $q$:

$$\frac{\partial p}{\partial q} = \frac{v'(u(\theta, \bar{q}))}{\phi(q, q; \bar{q}, \bar{q})} \cdot \frac{\partial u(\bar{q}, \bar{q})}{\partial \bar{q}}$$  \hspace{1cm} (6)$$

where $\phi(q, q; \bar{q}, \bar{q}) \equiv v' \left( v^{-1}(v(u(\bar{q}, q)) - v(u(\bar{q}, q)) + v(u(\bar{q}, q))) \right)$, which shows that the marginal quality effect on the H price may also depend on the L quality as $\phi$ involves $q$. The low-type’s taste parameter $\theta$ also affects the marginal quality effect of the H price. This finding has a crucial implication for the high-type quality: in the presence of narrow framing, the standard result of ‘no distortion at the top’ does not hold when the monopolist attempts to induce the narrow framing high-type consumers to buy the H menu as well. The optimal quality for the high-type is characterized by

$$c'(\bar{q}) = \frac{\partial p}{\partial q} = \frac{v'(u(\bar{q}, \bar{q}))}{\phi(q, q; \bar{q}, \bar{q})} \cdot \frac{\partial u(\bar{q}, \bar{q})}{\partial \bar{q}}.$$  \hspace{1cm} (7)$$

Hence, if $v'(u(\bar{q}, \bar{q}))/\phi(q, q; \bar{q}, \bar{q}) < 1$, the monopolist will choose a lower quality due to the narrow framing of consumers than the socially optimal quality which is the case without the narrow framing. We prove this result in Proposition 2 which builds on the intuition that the narrow framing consumer’s preferences tilt towards the L menu that has a higher quality/price ratio. Thus, in order to induce them to buy the original H menu with a higher quality and a higher price bundle, the monopolist needs to increase the H menu’s quality/price ratio, which calls for a distortion toward a relatively lower quality and lower price bundle compared to the original bundle.

**Proposition 2** (A distortion at the top from narrow framing). In the presence of narrow framing consumers, if the monopolist wants to induce the narrow framing consumers to buy the H menu $(\bar{q}, \bar{p})$ rather than the L menu $(q, p)$, even the optimal H menu quality $\bar{q}$ can deviate downward from the first-best quality.

*Proof.* See the Appendix.
Proposition 2 has at least two important implications for the optimal screening scheme. First, it shows how a behavioral bias by a subset of consumers can thwart the efficiency result that arises when this bias is not considered. Second, it shows a new type of externality in the sense that the narrow framing affects the choices available to other consumers: here the high-type neutral consumers cannot enjoy the first-best quality any more when a fraction of other high-type consumers have narrow frames.

Proposition 2 provides the existence of a downward distortion even at the top. It does not tell how the degree of narrow framing would affect the size of the distortion. We studied the comparative statics of the high quality with respect to the degree of narrow framing; we find that the distortion increases with the degree of narrow framing.

**Proposition 3** (Comparative statics for $q^*$ with the degree of narrow framing). Assume $u(\theta, q) = \theta q$. Then, the optimal H menu quality $q^*$ (when the monopolist induces the narrow framing consumers to buy the H menu) decreases with the degree of narrow framing.

*Proof.* See the Appendix.

Intuitively, the monopoly firm needs to offer a better quality-price ratio to the narrow framing consumers in order to keep them choosing the H menu. The higher the degree of narrow framing, the greater this adjustment is called for.

Regarding $q$, we have

$$\frac{\partial \pi}{\partial q} = (1 - \lambda) \left[ \frac{\partial u(\theta, q)}{\partial q} - c'(q) \right] + \lambda \frac{\partial \bar{p}}{\partial q} = 0 \quad (8)$$

which yields

$$c'(q) = \frac{\partial u(\theta, q)}{\partial q} + \frac{\lambda}{1 - \lambda} \frac{\partial \bar{p}}{\partial q} \quad (9)$$

In the absence of the framing effect, the downward distortion at the bottom arises due to the second term, which is proportional to the size of the information rent, $\frac{\partial \pi}{\partial q}$. Formally, the marginal effect of
the L quality on the H price without narrow framing is given by

\[- \frac{\partial \bar{p}}{\partial \bar{q}} = \frac{\partial u(\bar{\theta}, \bar{q})}{\partial \bar{q}} - \frac{\partial u(\theta, q)}{\partial q} = \Delta \theta \tag{10} \]

where the last equality will hold if \( u(\theta, q) = \theta q \). Thus, the key question is whether and how the narrow framing affects the information rent. Intuitively we just find that the distortion at the top arises under narrow framing. As the H quality drops from the first-best benchmark, the monopolist has less incentive to induce the downward distortion to prevent the high-type consumers from buying the L menu. This suggests that at the optimum the monopolist may choose a smaller degree of downward distortion at the bottom compared to the case of the neutral framing benchmark. Indeed, we can prove this claim mathematically. With some algebra and calculus, we can easily show (see the details in the Appendix) that

\[- \frac{\partial \bar{p}}{\partial \bar{q}} = \alpha \frac{\partial u(\bar{\theta}, \bar{q})}{\partial \bar{q}} - \beta \frac{\partial u(\theta, q)}{\partial q} \text{ where } \alpha < 1 \text{ and } \beta > 1 \tag{11} \]

when the narrow framing is taken into account. As the information rent decreases, the downward distortion is subsequently reduced.

**Proposition 4** (A distortion at the bottom from narrow framing). When the monopolist induces the high-type narrow framing consumers to buy the H menu \((\bar{q}, \bar{p})\) rather than the L menu \((q, p)\), the downward distortion at the L quality \(q\) decreases compared to the standard benchmark with no narrow framing.

**Proof.** See the Appendix. \( \square \)

What are the comparative statics of the low quality with respect to the degree of narrow framing? We find that a higher degree of narrow framing reduces the distortion.

**Proposition 5** (Comparative statics for \(q^*\) with the degree of narrow framing). Assume \( u(\theta, q) = \theta q \). Then, the optimal L menu quality \( q^* \) (when the monopolist induces the narrow framing con-
sumers to buy the H menu) increases with the degree of narrow framing.

**Proof.** See the Appendix.

This result needs to be understood jointly with the downward distortion at the top. As the monopolist introduces a downward distortion even for the premium menu to accommodate the narrow framing consumers, the incentive to degrade the basic menu to prevent the high-type from buying this basic menu has declined. As a result, the distortion arises in a smaller magnitude with narrow framing.

### 4.3 The Shutdown

The monopolist has another alternative of selling to the high-type consumers with a single menu. By doing so, the monopolist need not worry about the narrow framing consumers’ switching to the low-type menu. A necessary condition for the monopolist to choose to offer the high quality even to the low type consumers is that the low-type consumers’ marginal utility gain from the higher quality exceeds the marginal cost of providing the higher quality relative to the lower quality. Formally, \( u(\theta, q) - u(\theta, q^F) > c(q^F) - c(q) \).

Once the monopolist sells to the high-type consumers only, then his profit will be computed as \( \lambda [u(\theta, q^{FB}) - c(q^{FB})] \). The quality under the shutdown will be the first-best one with the full-extracting price \( p(q^{FB}) = u(\theta, q^{FB}) \). Alternatively, the monopolist can decide to sell to all consumers. Then, the participation of the L type consumers will bind and thus the profit for the monopolist will be equal to \( u(\theta, q^{FB}) - c(q^{FB}) \) because \( q = q^{FB} \) and \( p(q^{FB}) = u(\theta, q^{FB}) \). As a result, the shutdown is not optimal if

\[
\lambda < \lambda^o \equiv \frac{u(\theta, q^{FB}) - c(q^{FB})}{u(\theta, q^{FB}) - c(q^{FB})}.
\]
5 Application: Collateral as a Screening Device in Loan Markets

Thus far we have presented our key insights in the well-known context of non-linear price discrimination with optimal quality choice. In fact, the insights obtained in this environment may be extended to other relevant applications that involve asymmetric information between a principal who offers a menu of contracts to induce self-selection and a behavioral agent who is subject to narrow framing.

In this spirit we next consider a model of screening in credit markets when a bank (lender, principal) offers loan contracts to liquidity-constrained entrepreneurs (borrowers, agents) without knowing the probability of the entrepreneurs’ success on their investment projects. Specifically, we follow from Freixas and Rochet (2008) Section 4.6 which adapts Bester (1985) to a monopoly lender. The lender can offer different loan contracts with variable collateral requirements and the interest rate is a decreasing function of the collateral. After describing the model, deriving the first-best benchmark contracts where the lender is assumed to observe the borrower’s type, and identifying the second-best contracts where the type is the borrower’s private information, we study the effect of narrow framing and discuss its implication focusing on empirical predictions.\(^5\)

5.1 The Model

There are two types of entrepreneurs: a high-risk entrepreneur has a project with a lower success probability compared to a low-risk type. Let \(\theta_L\) and \(\theta_H\) denote their respective failure probability with \(0 < \theta_L < \theta_H < 1\). That is, the high-risk entrepreneur’s success probability, \(1 - \theta_H\), is lower than the low-risk type’s success probability, \(1 - \theta_L\).\(^6\) Let \(U_k\) denote the reservation utility (outside option) for risk-type \(k = H, L\). Assume \(U_L/(1 - \theta_L) > U_H/(1 - \theta_H)\): the low-risk type has a better outside option compared to the high-risk type. In this environment, the high-risk type’s

\(^5\)The notations and derivations in Section 6.1-3 are from ‘Lecture Note on Contract Theory’ by Jinwoo Kim(https://sites.google.com/site/jikim72/home); we do not claim any novelty in these parts. Our novel contribution starts from section 6.4 and on.

\(^6\)Recall that the key issue in the price discrimination application was to prevent the high-valuation consumer from mimicking the low-valuation consumer. In the loan market application, the essential part of the mechanism design is to prevent the high-risk borrowers from mimicking the low-risk borrowers.
incentive compatibility (IC) constraint and the low-risk type’s individual rationality (IR) constraint are binding when the self-selection is accordingly induced. Assume \((1 - \theta_k)y > U_k\) where \(y > 0\) is the return from the successful project, which means that each project \(k\) needs to be supported for social efficiency. Suppose a monopolistic lender offers a menu of loan contracts \(\{(C_k, R_k)\}_{k=L,H}\) where \(R_k\) and \(C_k\) denote the firm’s repayment amount in case of success and collateral for risk type \(k\) which is taken away by the lender in case of failure. There is a cost of converting a liquidated collateral into cash, which means that the first-best requires zero collateral in contracts. To be specific, the lender obtains \(\delta C_k\) with \(\delta < 1\) from the original collateral size \(C_k\). Let the proportion of risk type \(k\) be \(\beta_k\) where \(\beta_H = 1 - \beta_L\). The payoff of a risk type \(k\) borrower is

\[
(1 - \theta_k)(y - R_k) - \theta_k C_k
\]

for \(k = L, H\). The payoff of the lender is given by

\[
\Pi = \beta_H \left[ (1 - \theta_H)R_H + \theta_H \delta C_H \right] + \beta_L \left[ (1 - \theta_L)R_L + \theta_L \delta C_L \right] \tag{13}
\]

5.2 The First-Best Benchmark

Consider the full information benchmark in which the lender is able to observe the borrower’s risk type. Then at an optimal contract, the individual rationality constraints \(IR_k\) are binding: that is, \((1 - \theta_k)(y - R_k) - \theta_k C_k \geq U_k\) because \(\pi_k = (1 - \theta_k)R_k + \theta_k \delta C_k\) is maximized at

\[
\hat{C}_k = 0, \quad \hat{R}_k = y - \frac{U_k}{1 - \theta_k} \quad \text{for each} \ k.
\]

At the first-best contract (denoted by hat on the variables), there requires no collateral because the liquidation of collateral is costly. Since the low-risk type borrower has a better outside option than the high-risk type, the repayments will be set such that \(\hat{R}_H > \hat{R}_L\). If the lender cannot observe the borrower’s risk type, then the proposed contracts with no collateral would not work because the
high-risk type borrower will mimic the low-risk type and claim her risk type as $\theta_L$.

In Figure 3, we illustrate the first-best optimal contracts in the coordinate space of $(C, R)$. The indifference curves for risk type $k$ borrower have the flatter slope of $\frac{\theta_k}{1-\theta_k}$ compared to the slope of the iso-profit line for risk type $k$, $\frac{\delta \theta_k}{1-\theta_k}$. This means that the lender’s payoff will be maximized at $C_k = 0$.

### 5.3 Optimal Contracts with Asymmetric Information

Consider now the optimal menu of contracts when the lender does not know the borrower risk types. As usual, we apply the revelation principle (Myerson 1981) and then have two binding constraints. First, the lender needs to prevent the high-risk type from mimicking the low-risk type, which means that $IC_H$ must be binding at the optimal contract. And we have $C_H^* = 0$. To see why $C_H^* = 0$ must be the case at the optimal contract, we can consider what would happen with
If \( C_H > 0 \), then the lender can offer an alternative contract \( \{(C_L, R_L), (C'_H, R'_H)\} \) where \( C'_H = C_H - \varepsilon \) and \( R'_H = R_H + \frac{\theta_H}{1 - \theta_H} \varepsilon \) for small \( \varepsilon > 0 \). With this change, the lender’s profit goes up by \( \theta_H(1 - \delta)\varepsilon \) without any change in \( IC_H, IR_L, \) and \( IR_H \). Second, at the optimal contract the low-risk type’s participation constraint \( IR_L \) must be binding; otherwise, again the principal can find a profitable deviation. Based on this typical procedure, the lender’s problem is simplified to

\[
\max_{R_H, R_L, C_L} \Pi = \beta_H (1 - \theta_H) R_H + \beta_L [(1 - \theta_L) R_L + \theta_L \delta C_L]
\]

subject to

\[
(1) IC_H \quad (1 - \theta_H)(y - R_H) - \theta_H C_H = (1 - \theta_H)(y - R_L) - \theta_H C_L \tag{15}
\]
\[
(2) IR_L \quad (1 - \theta_L)(y - R_L) - \theta_L C_L = U_L \tag{16}
\]
\[
(3) IR_H \quad (1 - \theta_H)(y - R_H) \geq U_H \tag{17}
\]

By solving the two binding constraints (15) and (16), we obtain:

\[
C_L = \frac{(1 - \theta_L)(1 - \theta_H)}{\theta_H - \theta_L} R_H + a_1 \tag{18}
\]
\[
R_L = -\frac{\theta_L (1 - \theta_H)}{\theta_H - \theta_L} R_H + a_2 \tag{19}
\]

where \( a_1 = \frac{(1 - \theta_H)U_L - (1 - \theta_L) \gamma}{\theta_H - \theta_L} \) and \( a_2 = \frac{\theta_H (1 - \theta_L) \gamma - U_L}{\theta_H - \theta_L} \) are constant. Substituting (18) and (19) into the objective function (14), the lender’s revised objective function is given by

\[
\Pi = (1 - \theta_H) R_H \left[ \beta_H - \frac{\beta_L (1 - \delta)(1 - \theta_L) \theta_L}{\theta_H - \theta_L} \right] + a_3 \tag{14}
\]

where \( a_3 = \beta_L (1 - \theta_L) a_2 + \theta_L \delta a_1 \) is another constant. There are two different cases.

(i) If \( \Omega < 0 \), then it is optimal to decrease \( R_H \) as much as possible, which means that \( R_H^\ast = \hat{R}_L \). In this case, the optimal contract is characterized by a pooling contract \( (C_H, R_H) = (C_L, R_L) = \)
(0, \tilde{R}_L).

(ii) By contrast, if \( \Omega > 0 \), then it is optimal to increase \( R_H \) until \( IR_H \) is binding such that \((1 - \theta_H)(y - R_H) = U_H \). That is, \( R_H^* = \tilde{R}_H \), and the optimal contracts for \( C_L \) and \( R_L \) are obtained from (18) and (19) with \( R_H^* = \tilde{R}_H \). The optimal separating equilibrium is characterized by \((C_H^* = 0, R_H^* = \tilde{R}_H) \) and \((C_L^* > 0, R_L^* < \tilde{R}_L) \).

Focusing on the more interesting case (ii), we see that the lender offers the first-best contract with zero collateral to the high-risk type borrower. This is analogous to ‘no distortion at the top’ in the nonlinear price discrimination model. By contrast, the lender “distorts” the contract to the low-risk type borrower with a positive amount of collateral \((C_L^* > 0)\) with a smaller repayment amount compared to the first-best contract. Consequently, the lender uses different levels of collateral to screen the two different risk types of borrowers.

In Figure 3, we depict the second-best optimal contracts in the coordinate space of \((C, R)\). The high-risk type’s second-best contract \((C_H^*, R_H^*)\) is the same as its first-best one, \((0, \tilde{R}_H)\). The low-risk type’s second-best contract is determined by the intersection of the high-risk type’s indifference curve where \( IR_H \) is binding and the low-risk type’s indifference curve where \( IR_L \) is binding. Any contract on the line segment between \((0, \tilde{R}_L)\) and \((C_L^*, R_L^*)\) cannot be optimal because the high-risk type will deviate to such a contract.

5.4 Narrow Framing and Loan Contracts

We now introduce narrow framing into the model. Assume that a proportion \( \alpha \) of the high-risk type borrowers have narrow framing so that they prefer the L contract \((C_L, R_L)\) to the H contract \((C_H, R_H)\) when the neutral framing high-risk type borrowers are indifferent between the two. Since it is more profitable to keep the high-risk type borrowers at the given contract \((C_H = 0, R_H = \tilde{R}_H)\) than letting them switch to the L contract \((C_L^*, R_L^*)\), the lender will induce the high-risk type borrower to continue to buy the H menu. For this inducement, the incentive constraint of the
narrow framing high-risk type borrowers needs to be binding:

\[
IC_{H}^{\text{narrow}} : (1 - \theta_H) [v(y) - v(R_H)] - \theta_H v(C_H) = (1 - \theta_H) [v(y) - v(R_L)] - \theta_H v(C_L).
\]  

(20)

Then, the lender’s problem changes into

\[
\max_{R_H, R_L, C_H, C_L} \Pi = \beta_H [(1 - \theta_H)R_H + \theta_H \delta C_H] + \beta_L [(1 - \theta_L)R_L + \theta_L \delta C_L]
\]

subject to (20) and

\[
IR_L : (1 - \theta_L)(y - R_L) - \theta_L C_L = U_L,
\]

\[
IR_H : (1 - \theta_H)[v(y) - v(R_H)] \geq U_H.
\]

When there is no narrow framing, we must have had \( C_H^* = 0 \). We revisit whether this is still true when there is narrow framing and find \( C_H^n = 0 \) where the subscript \( n \) denotes the second-best optimal contract under narrow framing.

**Lemma 2.** The optimal second-best contract for the high-risk type narrow framing borrower is the same as the first-best result of no collateral, i.e., \( C_H^n = C_H^* = \hat{C}_H^* = 0 \).

*Proof.* See the Appendix.

As a result, we find that the first-best contract for the high-risk type borrowers is \( (C_H^n = 0, R_H^n = \hat{R}_H) \). However, we find that there is a change to the optimal contract offered to the low-risk type borrower due to the narrow framing.

**Proposition 6.** Suppose that there are high-risk type narrow framing borrowers and that the net social surplus generated from the project of the low-risk type borrower is greater than that of the high-risk type, \((1 - \theta_H)y - U_H < (1 - \theta_L)y - U_L\). Then, the optimal second-best contract for the low-risk type changes into a smaller collateral requirement and a higher repayment amount than the contract without the narrow framing, i.e., \( C_L^n < C_L^* \) and \( R_L^n > R_L^* \).
Proof. See the Appendix.

Proposition 6 implies the narrow framing of the high-risk type borrower generates externalities onto the low-risk type borrower by reducing the collateral requirement. The intuition is as follows. The low-risk type borrower’s IR constraint is binding and her contract is determined by the lender’s incentive to prevent the high-risk type borrower from having an incentive to mimic the low-risk type. Since the high-risk type narrow framing borrower perceives that the cost associated with collateral is much higher due to her narrow framing, now the marginal rate of substitution between collateral and interest rate becomes large for the low-type borrower. The lender thus can screen out the high-risk type borrower with a smaller collateral (as a screening instrument) from the low-risk type. Because collateral incurs social inefficiency due to its liquidation cost, the narrow framing improves social efficiency.\footnote{This result is similar to the finding in our price discrimination analysis that narrow framing does not necessarily lead to a loss in social welfare.}

In Figure 4, we have drawn the high-risk type narrow framing borrower’s indifference curve. As in Lemma 2, the marginal rate of substitution between $C$ and $R$—the slope of a tangent line to the narrow framing consumer’s indifference curve—is greater than the iso-profit line’s slope for the high-risk type. Moreover, as Proposition 6 shows, when $(1 - \theta_H)y - U_H < (1 - \theta_L)y - U_L$, the narrow framing borrower’s marginal rate of substitution between $C$ and $R$ is greater than that for the high-risk type neutral borrower in the area above the 45 degree line. Since the second best result $(C^*_L, R^*_L)$ lies above the 45 degree line if $(1 - \theta_H)y - U_H < (1 - \theta_L)y - U_L$, the second-best contract for the low-risk type borrower must be located at $(C^n_L, R^n_L)$. Figure 4 illustrates the result in Proposition 6, $C^n_L < C^*_L$ and $R^n_L > R^*_L$.

5.5 Empirical Predictions: Loan Markets and Narrow Framing

There are a variety of screening devices the bank uses in the credit market. Two major ones are interest rates and collateral. Stiglitz and Weiss (1981) and Wette (1983) show that utilizing only one screening device will not prevent adverse selection from occurring; Bester (1985) shows that
collateral and interest rates can be used simultaneously to attenuate adverse selection. Specifically, Bester points out a negative correlation between borrowers’ sensitivity to collateral and their default unobservables. In other words, high-risk type borrowers are inclined to choose the loan contracts with a higher interest rate and lower collateral, but low-risk type investors prefer the reversed ones. This prediction is consistent with the second-best optimal contracts described in section 6.3. Also, the prediction is supported by Crawford et al. (2018) and Ioannidou et al. (2018) who provide empirical evidence of a positive relationship between borrowers’ price sensitivity and their default rate.

The narrow framing effect captured by Proposition 6 implies that the low-risk type borrowers will be required to lay down a smaller collateral compared to the situation where narrow framing

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8There is also a large literature discussing how collateral and interest rates could minimize ex post frictions such as moral hazard (Boot and Thakor 1994), costly state verification (Gale and Hellwig 1985), and imperfect contract enforcement (Albuquerque and Hopenhayn 2004).
is absent. Notice that Bester (1987) finds that credit rationing can occur if the borrowers have a limited ability to provide the required level of collateral. Therefore, our first empirical prediction is that the smaller collateral requirement for the low-risk type borrowers due to narrow framing may reduce the credit rationing. Specifically, some borrowers with the collateral between $C^n_L$ and $C^*_L$ are no longer facing the credit rationing due to narrow framing.

Note that the smaller collateral means that the low-risk type borrowers face relatively higher interest rates than if the narrow framing effect was not taken into account. Thus, while the separation among heterogeneous borrowers is still implemented through a self-selection process, the difference between the two separating contracts will be smaller in the presence of narrow framing. In this sense, the model also predicts that the heterogeneity in loan contracts will decrease in a loan market where narrow framing is more prevalent.

From the lender’s perspective, the borrowers’ narrow framing reduces the lender’s monitoring cost. Several studies have documented that creditors use collateral as a complement to monitoring intensity. Rajan and Winton (1995) first theoretically present that pledging collateral improves the bank’s incentives to monitor. In accord with this result, Ono and Uesugi (2009) use a survey data set of Japanese small and medium enterprises to investigate this issue empirically. They show that the banks endogenously enhance the monitoring intensity to the borrowers whose loan claims are collateralized. In addition, Cerqueiro et al. (2016) demonstrate that an exogenous decrease in collateral values triggers higher loan rates, tighter credit limit and lower monitoring intensity. Overall, these studies report a positive correlation between the collateral and monitoring strength. In our paper, Proposition 6 shows that the collateral requirements for low-risk type investors become smaller under narrow framing. Given the relationship documented by these studies, our model suggests that the presence of narrow framing would induce the lender to monitor borrowers less intensively, other things being equal.
6 Conclusion

In this paper we study the classical problem of non-linear price discrimination when some agents engage in narrow framing. Narrow framing is a pervasive bias which leads to deviations from expected utility maximization in individual choice (Tversky and Kahneman 1981; Rabin and Weizsäcker 2009), and can help explain anomalous behaviors in financial markets (Benartzi and Thaler 1995; Barberis et al. 2006), insurance markets (Gottlieb and Mitchell 2015), and product markets (Schneider and Kim 2020). In this paper, we probed the implications of narrow framing for mechanism design.

Narrow framing is particularly relevant for principal-agent problems since narrow framers are biased toward options with higher quality-price ratios which influences the incentive compatibility constraints of the mechanism design problem. A principal who optimally responds to a narrow framing agent no longer preserves the classical result of no distortion at the top. Instead, the optimal contract reduces (but does not eliminate) the gap in quality offered to high and low type agents. We also find that narrow framing does not necessarily reduce total social welfare, relative to the standard case without narrow framing.

Although in the paper we offer key insights in the applications to non-linear price discrimination and to a lender’s screening problem where borrowers have heterogeneous credit risks, we expect that the general insights obtained from considering narrow framing agents in the optimal screening contracts appear to be relevant to other contexts as well. For instance, another promising application might be the design of an insurance contract.

Suppose a monopolistic insurer (principal) offers a menu of insurance packages to the potential insured (agents) who have different private types regarding their taste on the quality of insurance. Here the quality \( q_\theta \) with \( \theta \in \{ \bar{\theta}, \theta \} \) can be interpreted as various measures for the quality of insurance such as the scope of the insurance coverage or the maximum amount of coverage in an accident. The high-type agents have a stronger preference for insurance quality. The price \( p(q_\theta) \) would mean the ‘price’ of the given quality of insurance contracts such as the monthly insurance
premium. In this context, narrow framing may imply that some of the high-type insured agents are attracted to the contract with the higher quality-price ratio, which is indeed the low coverage and low price menu. When the insurer finds it better to induce the high-type narrow framing insured to choose his modified premium menu, there will arise quality adjustments not only for the premium menu but also for the basic one. As was the case for the price-discrimination model, the standard neutral high-type consumers possibly benefit from the narrow framing as their price may drop farther than the quality degradation. Our model also indicates that the new adjusted contracts may not always lead to a worse societal outcome. The insurance markets provide an interesting and important research environment where behavioral contract theory, particularly in the context of screening as well as moral hazard, will shed new insight on the design of optimal mechanisms.

There are several more directions for future research. One direction is to consider the effect of competition on the optimal screening contract (i.e., in markets with no monopolist). For instance, does competition mitigate the effects of narrow framing on the design of the optimal contract and on social welfare? Another avenue is to conduct tests of the primary empirical predictions of the model such as the narrowing of the gap in quality levels between consumers with high and low valuations for quality. A third avenue is to consider a setting with a continuum of agent types instead of the high and low types that we studied here. For instance, Severinov and Deneckere (2006) offers a tractable analysis for nonlinear pricing in a model with a continuum of types. In this article we focus on a simple two-type agents to highlight the key insight.

References


Appendices

A Numerical Illustrations for Optimal Choices and Welfare Comparison

In this Appendix, we study the monopolist’s profit-maximizing choice among the three alternatives: no inducement, inducement and shutdown. We first characterize the general principles for the optimal choice and then provide numerical exercises with parametric assumptions for better illustrations of our general results (Propositions 2-5). Furthermore, we discuss how consumer surplus and social welfare changes with the degree of narrow framing.

A.1 Alternatives

As we mentioned earlier, the monopolist chooses the most profitable option among the three options of (i) shutting down the low type consumers and serving only the high type with the first-best quality, (ii) inducing the narrow framing high type consumers to buy the same H menu with the neutral framing high type consumers, or (iii) giving up the inducement and following the standard second-degree mechanism design accepting that the high-type narrow framing consumers will choose the L menu.

The maximum profit from the shutdown option is given by

\[ \pi_S = \lambda [u(\theta, q^{FB}) - c(q^{FB})] \]  

where the subscript \( S \) in \( \pi \) stands for the shutdown. The high-type neutral framing consumers earn the net surplus of \( u(\theta, q^{FB}) - p^{FB} = 0 \). Note that the high-type narrow framing consumers no longer have an alternative offer to deviate; they will buy the H menu at \( v(u(\theta, q^{FB})) - v(p^{FB}) = 0 \), which is always satisfied under \( u(\theta, q^{FB}) - p^{FB} = 0 \). There can be a debate how one measures the narrow-framing consumers’ welfare. Schneider and Kim (2020) argue that from the social perspective, the surplus should be measured through neutral frames. As illustrated by empirical violations of stochastic dominance and other consumer choices studied in Schneider and Kim (2020), narrow framing can lead to choices that are dominated in an objective sense. As Hilgers and Wibral (2014) conclude from an experimental study on narrow bracketing (what we refer to as narrow framing), “Our results further suggest that narrow bracketing is a mistake rather than the outcome of rational maximization of preferences” (p.1). Narrow frames thus do not seem be the proper yardstick for evaluating social welfare. In this regard, from the perspective of welfare evaluation, the high-type narrow framing consumers also have the true surplus of \( u(\theta, q^{FB}) - p^{FB} \), which is equal to zero, and is the same as the surplus for the neutral framing consumers.

Second, the monopolist may induce the high-type narrow framing consumers to purchase the
premium menu \((\bar{q}, \bar{p})\). With the inducement scheme, the monopolist’s profit is given by

\[
\pi_I = \lambda [p^* - c(\bar{q}^*)] + (1 - \lambda) [p^* - c(q^*)]
\]  

(A.2)

where \(q^*\) and \(q^*\) are the second-best optimal qualities characterized as in Section 4.2. We have already shown \(\bar{q}^* \leq \bar{q}^{FB}\) (a downward distortion even at the top) and \(\bar{q}^{SB} < q^* < \bar{q}^{FB}\) (a reduced downward distortion at the bottom). Compared to the shutdown, the monopolist earns less from the high-type consumers because \(\bar{q}^* < \bar{q}^{FB}\). Instead, there are additional profits from the low-type consumers, which can yield a greater overall profit compared to the shutdown.

Third, the monopolist can choose to not induce the narrow framing consumers. With this choice, he can remove the quality distortion at the top. Let the double asterisk (**) denote the optimal qualities with no inducement. Then, we obtain \(\bar{q}^{**} = \bar{q}^{FB}\) which yields a higher mark-up per a high-type neutral consumer. By contrast, since now the high-type narrow framing consumers buy the less profitable menu \((p^{**}, q^{**})\) along with the low-type consumers, some mark-up losses arise from the group of the high-type consumers. The profit under no inducement is given by

\[
\pi_N = \lambda (1 - \alpha) [p^{**} - c(q^{FB})] + (\lambda \alpha + (1 - \lambda)) [p^{**} - c(q^{**})]
\]  

(A.3)

where \(p^{**} = u(\theta, \bar{q}^{FB}) - (u(\theta, q^{**}) - u(\theta, q^{**}))\) and \(p^{**} = u(\theta, q^{**})\).

In principle, the monopolist’s optimal decision is the one that gives the highest profit, i.e., \(\max\{\pi_S, \pi_N, \pi_I\}\). Because a consumer will always have \(u(\theta, q) - p \geq 0\) (IR constraint) and the seller will have a non-negative profit \(p(\theta) - c(q(\theta)) \geq 0\) from each customer (otherwise no sales are made), the shutdown choice will not be the socially optimal choice. In addition, if (12) holds, the monopolist will not choose the shutdown either. Therefore, we can focus on the two choices, inducement and no inducement, for the monopolist and for the social planner.

Ideally, one may want to fully characterize the monopolist’s optimal choice and compute the social welfare at the optimal quality. However, such a general comparison and characterization are not feasible with the general indirect utility function \(u\) and the general prospect theory value function \(v\) and the general cost function \(c\), because the profits change over many parameters and functional forms. Since the general expression of the profit and social welfare do not allow for an explicit analysis of the optimal choices, in the following subsection, we take a more practical approach by adopting specific functional forms with the aid of numerical exercises.
A.2 Numerical Exercises with Specific Functions

Here we assume the linear indirect utility function of \( u(\theta, q) = \theta q \), which gives \( \partial u / \partial q = \theta \) and a standard power prospect theory value function \( v \) introduced in

\[
v(x) = \begin{cases} 
  x^\rho, & \text{for } x \geq 0 \\
  -|x|^\rho, & \text{for } x < 0 
\end{cases}
\]  

(A.4)

If \( \rho = 1 \), narrow framing coincides with neutral framing. For the cost function, we assume a simple quadratic function of \( c(q) = \frac{1}{2} q^2 \), which gives \( c'(q) = q \). With this specification, the first-best qualities are given by \( q^{FB} = \overline{\theta} \) and \( q^{FB} = \theta \). The second-best qualities with no narrow framing (which is equivalent to \( \rho = 1 \)) are given by \( q^{SB} = \overline{\theta} \) and \( q^{SB} = \theta - \frac{\lambda}{1-\lambda} \Delta \theta \).

A.2.1 Optimal choices with different levels of spread, \( \Delta \theta \)

First, we examine how the monopolist’s optimal choices change with different levels of narrow framing (that is, with different values of \( \rho \)) under different levels of spreads \( \Delta \theta \). We fix

\[
\overline{\theta} = 3/2, \quad \lambda = 1/3, \quad \alpha = 1/2
\]

and consider different spreads with four different values

\[
\theta = 0.50, \quad 1.00, \quad 1.25, \quad 1.49.
\]

To report the summary of the numerical exercises\(^9\), we find that

(i) (Large spread: Shutdown optimal for all \( \rho \in [0, 1] \))

With \( \theta = 0.5 \) the monopolist always finds the shutdown optimal as it dominates both the inducement regime and the choice of no inducement in profits, for the entire range of \( \rho \in [0, 1] \).

As the asymmetric information on the type parameters plays a substantial role in affecting the qualities and subsequently the profit, by shutting down the low-type consumers the seller can reduce any profit loss from the quality distortion from serving the low-type consumers and prevent the high-type narrow framing consumers from consuming the low-type menu.

(ii) (Modest spread: Shutdown optimal for \( \rho \in [0, 0.838] \))

With \( \theta = 1 \) we find that the shutdown stays as the monopolist’s optimal choice when \( \rho < 0.838 \), but then inducement emerges as the optimal choice when \( \rho > 0.838 \). Intuitively, the

---

\(^9\)Detailed results of numerical exercises are available from the authors upon request.
higher degree of narrow framing due to the smaller $\rho$ makes it more costly for the monopolist to adopt the inducement regime because compensating for the narrow framing causes a greater quality distortion at the top, which could have been avoided under the shutdown strategy.

(iii) (Small spread: No Inducement optimal for $\rho \in [0, 0.631)$, Inducement optimal for $\rho \in (0.631, 1]$)

With $\theta = 1.25$ we find that the shutdown strategy does not arise any more as an optimal choice. Instead, the monopolist finds it better to choose the inducement regime only when $\rho > 0.631$, but switches to no inducement when $\rho < 0.631$ as inducement becomes a more expensive strategy. The shutdown strategy is dominated by the other choices because the spread is sufficiently small such that it is less attractive to not serve the low-type consumers.

(iv) (Negligible spread: Inducement optimal for all $\rho \in [0, 1]$)

With $\theta = 1.49$, the monopolist finds it optimal to choose the inducement regardless of $\rho$. As the two types of consumers exhibit almost identical tastes for quality, the narrow framing no longer plays a substantial role in the quality choices. The inducement dominates no inducement. This is because the benefit of no inducement, which arises from allowing the monopolist to achieve the first-best quality at the top, becomes very small, whereas the mark-up from the premium menu from the inducement is large enough compared to the mark-up from the less expensive menu with no inducement.

We think that case (iii) involves the most common and interesting choices between no inducement (Section 4.1) and inducement (Section 4.2), assuming away the shutdown. In this spirit, we examine case (iii) in more detail in the next subsection. Figure 1-(a) illustrates the monopolist’s optimal choice over the range of the degree of narrow framing. It shows that selling the premium H menu to the narrow framing consumers is optimal for $\rho \in (0.631, 1]$, but the optimal choice changes to no inducement when $\rho$ drops further down from the cutoff value, $\rho = 0.631$ as we described in case (iii).

A.2.2 Comparative statics for optimal quality schedules with narrow framing

Recall that Proposition 2 shows that the second-best quality at the top with the inducement choice, $q^*$, is subject to a downward distortion ($\overline{q}^* \leq \overline{q}^\text{FB}$) and that Proposition 3 establishes that such a downward distortion at the top increases with the degree of narrow framing. In Figure A.1-(b), we draw the monopolist’s quality schedules $\overline{q}^*(\rho)$ and $\underline{q}^*(\rho)$ over $\rho \in [0.5, 1]$ with $\rho = 1$
Figure A.1: Monopoly optimal regime and quality choices with narrow framing

![Graph](image)

(a) Optimal choices and profits

(b) Optimal qualities

being at the origin of the x-axis. The qualities at $\rho = 1$ represent the first-best benchmark qualities:

$$q^*(\rho = 1) = q^{FB} = \theta = 1.5; \quad q^*(\rho = 1) = q^{FB} = \theta = 1.25$$

We can see $q^* \leq q^{FB}$ under the optimal inducement choice while $q^* = q^{FB}$ holds under the optimal no inducement regime and the quality distortion itself gets bigger as the degree of narrow framing increases.

Regarding $q^*$, as Proposition 4 proves, we have $q^* \leq q^{FB}$ when the narrow framing consumers buy the H menu. Under the no inducement regime, there is still a downward distortion at the bottom. According to Proposition 5, the downward distortion decreases in its size as the degree of narrow framing increases, which is illustrated in Figure A.1-(b). Intuitively, the narrow framing consumers desire to have a greater quality-price ratio from the H menu and their IC constraint is binding, which means that the necessity of the downward distortion—which was created to minimize the information rent—has decreased. On the other hand, the downward distortion is larger under the inducement choice compared to the distortion under no inducement. This suggests that the narrow framing itself does not reduce the extent of the downward distortion at the bottom.
A.2.3 Comparative statics for consumer and social surplus with narrow framing

Another important comparative statics analysis pertains to how consumer surplus and social welfare vary with the narrow framing. Figure A.2-(a) illustrates that the high-type consumers earn higher surplus with the higher degree of narrow framing under the inducement regime. Once the monopolist chooses no inducement, then the quality for the high type is set to the first-best level and thus their surplus is independent of the level of $\rho$. Notice that the consumer surplus for a high-type consumer is not always higher under inducement compared to their surplus under no inducement. While the quality at the top decreases with the narrow framing, the price offered to the high-type consumers becomes much less expensive in order to induce the narrow framing consumers. Thus, it is possible to have a case where the monopolist still finds it better to induce the high-type narrow framing consumers who can benefit from the aggressive inducement offer compared to no inducement with first-best quality as the information rent under no inducement can be very small.

One caveat is that we do not claim that it is always the case that the high-type consumers benefit from a greater degree of narrow framing. In principle, the quality affects the consumer surplus through two channels: (i) the decreasing quality decreases the surplus (direct quality effect), which hurts the high-type consumers; (ii) the decreasing quality leads to a lower price, which benefits the
consumers (indirect price effect). Since the optimal quality depends on the firm’s cost $c(q)$, our numerical example cannot be interpreted to identify which of the two effects is generally greater.

Figure A.2-(b) shows how the social welfare, which consists of the consumer surplus and the firm’s profit, varies with narrow framing. First, it shows that the inducement regime can lead to a greater level of social welfare compared to the level of social welfare under no inducement. Second, since the firm’s profit decreases with the smaller $\rho$ while the high-type consumers earn higher surplus with the smaller $\rho$, the overall effect of the degree of narrow framing on social welfare depends on the relative magnitude of these two opposing forces. In the given example, the welfare increases with a higher degree of narrow framing. Thus, we can say that narrow framing does not necessarily hurt the social welfare.
B Mathematical Proofs

Proof of Proposition 2

Recall the inverse function theorem, \( v^{-1} = 1/v'(v^{-1}) \). Thus, the condition \( \phi(\bar{q}, q; \bar{\theta}, \theta) < 1 \) is equivalent to

\[
\begin{align*}
\frac{v'(u(\bar{\theta}, \bar{q}))}{v'(v^{-1}(v(u(\bar{\theta}, \bar{q})) - v(u(\bar{\theta}, q)) + v(u(\bar{\theta}, q)))} & \leq u(\bar{\theta}, \bar{q}) > v^{-1}(v(u(\bar{\theta}, \bar{q})) - v(u(\bar{\theta}, q)) + v(u(\bar{\theta}, q))) : v'' < 0 \\
& \leq v(v(u(\bar{\theta}, \bar{q})) > v(u(\bar{\theta}, \bar{q})) - v(u(\bar{\theta}, q)) + v(u(\bar{\theta}, q))) : v' > 0 \\
& \leq u(\bar{\theta}, \bar{q}) > u(\bar{\theta}, \bar{q})
\end{align*}
\]

QED

Proof of Proposition 3

From F.O.C. concerning \( \bar{q} \), we have

\[
c'(\bar{q}) = \frac{v'(u(\bar{\theta}, \bar{q}))}{\phi(\bar{q}, q; \bar{\theta}, \theta)} \frac{d\phi(u(\bar{\theta}, \bar{q}))}{d\bar{q}} \tag{B.1}
\]

where \( v'(u(\bar{\theta}, \bar{q})) / \phi(\bar{q}, q; \bar{\theta}, \theta) < 1 \).

Consider a concave function, \( \psi \), such that \( v_2(u(\bar{\theta}, q_2^*)) = \psi(v_1(u(\bar{\theta}, q_1^*))) \) where \( v_1 \) and \( v_2 \) stand for two different degrees of narrow framing. Since \( v_2 \) is a concave transformation of \( v_1 \), consumers with \( v_2 \) are more narrow framed than those with \( v_1 \). If \( u(\theta, q) = \theta q \), then \( c'(q_1^*) > c'(q_2^*) \iff q_1^* > q_2^* \). Since \( c'(q_1^*) > c'(q_2^*) \iff \phi_2 < \phi_1 \), below we show how \( \phi_2 < \phi_1 \) is established.

Since \( v_2(u(\bar{\theta}, q_2^*)) = \psi(v_1(u(\bar{\theta}, q_1^*))) \), we have

\[
\phi_2 = \frac{v_2'(u(\bar{\theta}, q_2^*))}{v_2'(v_1^{-1}(\psi(v_1(u(\bar{\theta}, q_1^*))) - \psi(v_1(u(\bar{\theta}, q_1^*))) + \psi(v_1(u(\bar{\theta}, q_1^*)))) + \psi(v_1(u(\bar{\theta}, q_1^*)))})
\]

For notation simplicity, let \( v_1(u(\bar{\theta}, q_1^*)) = x, v_1(u(\bar{\theta}, q_2^*)) = y, \) and \( v_1(u(\bar{\theta}, q_1^*)) = z \) where \( x < y < z \). The high-type narrow framing consumers’ IC constraint requires \( x - y = y - z \). For all \( x, y, z \) with \( x - y = y - z \), the concavity of \( \psi \) establishes the relationship of

\[
\psi(z) + \psi(x) < \psi(y) + \psi(z - y + x)
\]
\[ \iff v_2(\overline{p}_2) = \psi(z) - \psi(y) + \psi(x) < \psi(z - y + x) = \psi(v_1(\overline{p}_1)) \]

Since \( v_2(\overline{p}_2) < \psi(v_1(\overline{p}_1)) \), \( v_2(u(\overline{\theta}, q^*_2)) = \psi(v_1(u(\overline{\theta}, q^*_1))) \) and \( v'' < 0 \), we have

\[
\phi_2 = \frac{v'_2(u(\overline{\theta}, q^*_2))}{v'_2(\overline{p}_2)} = \frac{\psi'(v_1(u(\overline{\theta}, q^*_1))) \cdot v'_1(u(\overline{\theta}, q^*_1))}{v'_2(\overline{p}_2)} \quad : \quad \because v_2(u(\overline{\theta}, \overline{q}^*_2)) = \psi(v_1(u(\overline{\theta}, \overline{q}^*_1)))
\]

\[
< \frac{\psi'(v_1(u(\overline{\theta}, \overline{q}^*_1))) \cdot v'_1(u(\overline{\theta}, \overline{q}^*_1))}{\psi'(v_1(\overline{p}^*_1)) \cdot v'_1(\overline{p}^*_1)} \quad : \quad \because v_2(\overline{p}^*_2) < \psi(v_1(\overline{p}^*_1)) \Rightarrow v'_2(\overline{p}^*_2) > \psi'(v_1(\overline{p}^*_1)) v'_1(\overline{p}^*_1)
\]

\[
< \frac{v'_1(u(\overline{\theta}, \overline{q}^*_1))}{v'_1(\overline{p}^*_1)} = \phi_1 \quad : \quad \because v_1(u(\overline{\theta}, \overline{q}^*_1)) > v_1(\overline{p}^*_1) \Rightarrow \frac{\psi'(v_1(u(\overline{\theta}, \overline{q}^*_1)))}{\psi'(v_1(\overline{p}^*_1))} < 1
\]

QED

**Proof of Proposition 4**

Recall that \( \overline{p} = v^{-1}(v(u(\overline{\theta}, \overline{q})) - v(u(\overline{\theta}, q)) + v(u(\overline{\theta}, q))) \). The derivative of the price \( \overline{p} \) with respect to \( q \) is given by

\[
- \frac{\partial \overline{p}}{\partial q} = \alpha \frac{\partial u(\overline{\theta}, q)}{\partial q} - \beta \frac{\partial u(\overline{\theta}, q)}{\partial q}
\]

where

\[
\alpha = \frac{v'(u(\overline{\theta}, q))}{v'(v^{-1}(v(u(\overline{\theta}, q)) - v(u(\overline{\theta}, q)) + v(u(\overline{\theta}, q))))}
\]

and

\[
\beta = \frac{v'(u(\overline{\theta}, q))}{v'(v^{-1}(v(u(\overline{\theta}, q)) - v(u(\overline{\theta}, q)) + v(u(\overline{\theta}, q))))}.
\]

To prove that the downward distortion decreases due to the narrow framing, one sufficient condition is to have \( \alpha < 1 \) and \( \beta > 1 \). First, we can show that \( \alpha < 1 \) is equivalent to the followings:

\[
v'(u(\overline{\theta}, q)) < v'(v^{-1}(v(u(\overline{\theta}, q)) - v(u(\overline{\theta}, q)) + v(u(\overline{\theta}, q))))
\]

\[
\iff u(\overline{\theta}, q) > v^{-1}(v(u(\overline{\theta}, q)) - v(u(\overline{\theta}, q)) + v(u(\overline{\theta}, q))) \quad : \quad v'' < 0
\]

\[
\iff v(u(\overline{\theta}, q)) > v(u(\overline{\theta}, q)) - v(u(\overline{\theta}, q)) + v(u(\overline{\theta}, q))
\]

\[
\iff 2v(u(\overline{\theta}, q)) > v(u(\overline{\theta}, q)) + v(u(\overline{\theta}, q))
\]

where the last inequality holds when \( v \) is a concave function under

\[
u(\overline{\theta}, q) - u(\overline{\theta}, q) \geq u(\overline{\theta}, q) - u(\overline{\theta}, q)
\]

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which means that the information rent yielded to the high-type neutral consumers (LHS) is greater than the high-type’s gain from deviation to the L menu.

Second, \( \beta > 1 \) is equivalently given as

\[
v'(u(\theta, q)) > v'(v^{-1}(v(u(\theta, q)) - v(u(\theta, q))) + v(u(\theta, q)))
\]

\[
\iff u(\theta, q) < v^{-1}(v(u(\theta, q)) - v(u(\theta, q))) + v(u(\theta, q))
\]

\[
\iff v(u(\theta, q)) < v(u(\theta, q)) - v(u(\theta, q)) + v(u(\theta, q))
\]

\[
\iff v(u(\theta, q)) < v(u(\theta, q))
\]

\[
\iff u(\theta, q) < u(\theta, \bar{q})
\]

where the last inequality also holds because \( u \) is an increasing function of \( q \) and we have \( q < \bar{q} \).

QED

**Proof of Proposition 5**

From F.O.C. concerning \( q \), we have

\[
c'(q) = \frac{\partial u(\theta, q)}{\partial q} - \frac{\lambda}{1 - \lambda} \left( \alpha \frac{\partial u(\theta, q)}{\partial q} - \beta \frac{\partial u(\theta, q)}{\partial q} \right)
\]

where

\[
\alpha = \frac{v'(u(\theta, q))}{v'(v^{-1}(v(u(\theta, q)) - v(u(\theta, q))) + v(u(\theta, q)))}
\]

\[
\beta = \frac{v'(u(\theta, q))}{v'(v^{-1}(v(u(\theta, q)) - v(u(\theta, q))) + v(u(\theta, q)))}
\]

Let the two optimal qualities schedules denoted by \( q^*_1 \) and \( q^*_2 \) corresponding to the two different degree of narrow framing. That is,

\[
c'(q^*_1) = \frac{\partial u(\theta, q^*_1)}{\partial q^*_1} - \frac{\lambda}{1 - \lambda} \left( \alpha_1 \frac{\partial u(\theta, q^*_1)}{\partial q^*_1} - \beta_1 \frac{\partial u(\theta, q^*_1)}{\partial q^*_1} \right)
\]

\[
c'(q^*_2) = \frac{\partial u(\theta, q^*_2)}{\partial q^*_2} - \frac{\lambda}{1 - \lambda} \left( \alpha_2 \frac{\partial u(\theta, q^*_2)}{\partial q^*_2} - \beta_2 \frac{\partial u(\theta, q^*_2)}{\partial q^*_2} \right)
\]
Assume \( u(\theta, q) = \theta q \). If we show \( \alpha_1 > \alpha_2 \) and \( \beta_1 < \beta_2 \), then \( c'(q_2^*) > c'(q_1^*) \). We compare \( \alpha_1 \) and \( \alpha_2 \):

\[
\alpha_2 = \frac{v_2'(u(\overline{\theta}, q_2^*))}{v_2'(p_2^* )} < \frac{v_2'(u(\overline{\theta}, q_2^*))}{\psi'(v_1(p_1^* ))} \cdot v_1'(p_1^*) = \psi'(v_1(p_1^*))v_1'(p_1^*)
\]

\[
= \frac{\psi'(v_1(u(\overline{\theta}, q_1^* )))}{\psi'(v_1(p_1^*))} - \alpha_1
\]

Since we assume the information rent yielded to the high type neutral consumers is greater than the high type’s gain from deviation to the low menu, which means

\[
u(\overline{\theta}, q^*) - u(\theta, q^*) \geq u(\overline{\theta}, q^*) - u(\theta, q^*)
\]

\[
\iff 2v(u(\overline{\theta}, q^*)) \geq v(u(\overline{\theta}, q^*)) + v(u(\theta, q^*))
\]

\[
\iff v(u(\overline{\theta}, q^*)) \geq v(u(\overline{\theta}, q^*)) - v(u(\theta, q^*)) + v(u(\theta, q^*))
\]

\[
\iff v(u(\overline{\theta}, q^*)) \geq v(p^*) \quad \text{(B.3)}
\]

The inequality in the last line results in \( \alpha_1 > \alpha_2 \), because of the concavity of \( \psi(\cdot) \).

Next, we compare \( \beta_1 \) and \( \beta_2 \) in a similar manner:

\[
\beta_2 = \frac{v_2'(u(\theta, q_2^*))}{v_2'(p_2^* )} < \frac{v_2'(u(\theta, q_2^*))}{\psi'(v_1(p_1^* ))} \cdot v_1'(p_1^*) = \psi'(v_1(p_1^*))v_1'(p_1^*)
\]

\[
= \frac{\psi'(v_1(u(\theta, q_1^* )))}{\psi'(v_1(p_1^*))} - \beta_1
\]

Since \( u(\theta, q_1^*) = p_1^* < p_1^* \), it leads to \( \beta_2 > \beta_1 \). Therefore, we show \( c'(q_2^*) > c'(q_1^*) \iff q_2^* > q_1^* \).

QED
Proof of Lemma 2

The indifference curve of the narrow framing high-risk type borrower is:

\[(1 - \theta_H)[v(y) - v(R_H)] - \theta_H v(C_H) = v(U_H)\]  

(B.4)

The marginal rate of substitution of \(C\) and \(R\), which is (the absolute value of) the slope of the indifference curve (B.4), evaluated at \(C_H = 0\) is greater than the iso-profit line for \(\theta_H\):

\[-\left|\frac{dR_H}{dC_H}\right|_{C_H=0} = \frac{\theta_H v'(0)}{(1 - \theta_H)v'(R_H)} > \frac{\delta \theta_H}{1 - \theta_H}.

This is because \(v(\cdot)\) is a concave function and thus \(v'(0) > v'(R_H)\). Therefore, \(C_H^* = 0\). QED

Proof of Proposition 6

We show \(C_H^* < C_L^*\) in the following three steps.

First, we know that the indifference curve of the high-risk type narrow framing borrowers is a monotonically decreasing (Lemma 1). And, its slope of the marginal rate of substitution of \(C\) and \(R\) is steeper than the indifference curve of the high-risk type neutral framing borrower, i.e.,

\[-\frac{dR_H}{dC_H} = \frac{\theta_H v'(C_H)}{(1 - \theta_H)v'(R_H)} > \frac{\theta_H}{1 - \theta_H}\]  

(B.5)

at the point of \((0, \hat{R}_L)\). However, the indifference curve is convex up:

\[\frac{d^2 R_H}{dC_H^2} = -\frac{\theta_H v''(C_H)(1 - \theta_H)v'(R_H(C_H)) - \theta_H v''(C_H)(1 - \theta_H)v'(R_H)}{(1 - \theta_H)v'(R_H)^2} > 0\]

since \(v'' < 0\) and \(\frac{dR_H}{dC_H} < 0\). That is, the indifference curve becomes flatter as \(C\) increases.

Second, from (B.5) we can see that the given inequality holds for any pair of \((C_H, R_H)\) with \(C_H < R_H\). Hence, if the second-best contract under neutral framing, \((C_L^*, R_L^*)\), lies above the 45 degree line, then we have \(C_H^* < C_L^*\). This is because the indifference curve of the narrow framing borrower starting at \((0, \hat{R}_L)\) cannot pass the indifference curve for the neutral framing \(\theta_H\); to do so, it requires \(-\frac{dR_H}{dC_H} < \frac{\theta_H}{1 - \theta_H}\) which cannot be true for \(C_H < R_H\).

Third, we find the sufficient condition for \(C_L^* < R_L^*\). From (18) and (19), we derive:

\[(\theta_H - \theta_L)C_L^* = (1 - \theta_H)(1 - \theta_L)R_H + (1 - \theta_H)(U_L - (1 - \theta_L)y)\]
\[(\theta_H - \theta_L)R_L^* = -(1 - \theta_H)\theta_LR_H - \theta_H(U_L - (1 - \theta_L)y)\]
which leads to

\[(\theta_H - \theta_L)(C^*_L - R^*_L) = (1 - \theta_H)R_H + U_L - (1 - \theta_L)y.\]  \hspace{1cm} (B.6)

Since \(\theta_H - \theta_L > 0\), we have \(C^*_L < R^*_L\) if the RHS of (B.6) takes on a negative value:

\[(1 - \theta_H)R_H < (1 - \theta_L)y - U_L.\]  \hspace{1cm} (B.7)

Since the maximum value of \(R_H\) is attained at \(R_H = \hat{R}_H\), by substituting \(R_H = \hat{R}_H = y - \frac{U_H}{1 - \theta_H}\) we can derive the following sufficient condition:

\[0 < (1 - \theta_H)y - U_H < (1 - \theta_L)y - U_L.\]

Therefore, when the low-risk type’s social efficiency is larger than the high-risk type’s social efficiency, we have \(C^*_L < C^*_L\). QED